Evaluation of the Modulus of Elasticity of Wheat Grain[1]

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ABSTRACT

The mechanical properties of Seneca wheat grains subjected to uniaxial compression was studied with the Instron testing machine. Whole grains were loaded by means of parallel plates, a smooth spherical indenter, and a cylindrical indenter. Also, core specimens, prepared by cutting off both ends of the grains, were loaded by means of parallel plates. Under a constant rate of deformation of 0.020 in./min., the load-deformation relation in all four compression tests was linear up to a certain load, nonlinear beyond it. Cyclical loading-unloading to a low constant load within the linear portion of the load-deformation curve showed that the deformation was partly recovered and partly residual. The residual deformations gradually decreased to small constant values in the third cycle, whereas the elastic deformations remained constant. Under these conditions the behavior of the wheat grains was considered approximately Hookean, and the classic theory of elasticity was adapted for evaluation of their modulus of elasticity. Hertz's solution for convex bodies was used for compression of the whole grain by means of parallel plates and by the spherical indenter, and Boussinesq's solution for semi-infinite bodies subjected to concentrated compressive loads was used for the cylindrical indenter. Four values for the apparent modulus of elasticity were thus obtained for Seneca wheat grains of 9.1% moisture content, ranging from 1.6×10^5 to 8.3×10^5 p.s.i.

The mechanical properties of the individual wheat grain have been studied to a certain extent, mainly with respect to the hardness of the grain. With different types of testers (1–5), hardness values were obtained which serve their purpose for comparative studies but add little to the knowledge of basic mechanical properties of the individual grain.

So far very little has been reported on the rheological parameters of wheat grain. Shpolyanskaya (6) performed compression tests with static and impact loading, using a "laboratory impact tester." The wheat grain was considered a single "structural unit," disregarding its nonhomogeneous composition. Two types of wheat were studied under a uniaxial compression. For determination of the modulus of elasticity, Hertz's method for contact stress was applied; in this method the grain is considered to be a solid sphere. On the basis of the first loading cycle, a "modulus of deformability" was introduced which takes into consideration the total deformation. This modulus is claimed to be of practical significance, "since it reflects the mechanical properties of the grain in the form in which it arrives at the rollers of the first milling system." The author reports values for destructive force, modulus of elasticity, and deformability, as well as specific viscosity and other mechanical properties for two types of wheat at two moisture levels.

Zoerb and Hall (7) report a study of the basic mechanical and rheologi-

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cal properties of wheat grains, in which a load cell testing unit with strain gages was used as the sensing device. Compressive tests were conducted on "core specimens" of wheat, prepared by cutting off each end of the grain. The cross-sectional area of the core specimens was measured and the compressive stress and modulus of elasticity were determined.

The objective of the present work was to test the applicability of four different methods for evaluation of the modulus of elasticity of the individual wheat grain, as well as to study its general behavior under compressive loads.

MATERIAL AND METHODS

Seneca wheat equilibrated at 50% relative humidity and 72°F. was used for the experiments. Its moisture content was 9.10% (10% dry basis); it contained (moisture-free basis) protein, 12.55%; crude fiber, 3.43%; and ash, 1.72%. The mechanical properties of the individual wheat grains were determined by an Instron table model testing machine with a 50-lb. load cell. Four testing techniques (Fig. 1) were employed:

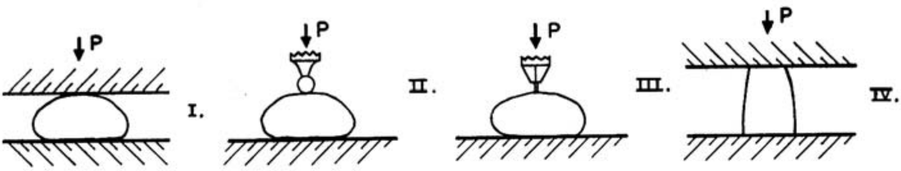


Fig. 1. Four methods of uniaxial compression of wheat grain: I, parallel plate (whole grain); II, spherical indenter; III, cylindrical indenter; IV, parallel plate (core specimens).

I. Uniaxial Compression between Two Parallel Flat Plates

Whole grains, crease side down, were fixed to a metal plate by means of a thin layer of DuPont Duco cement to prevent movement during the testing. A compressive load was applied by means of parallel plates.

Hertz's method for contact stresses between two elastic bodies subjected to uniaxial compression (8), as reviewed by Kozma and Cunningham (9) and Mohsenin (10), was employed for calculation of the modulus. According to this method, the deformation of the two convex bodies is given by

$$D = \frac{K}{2}\left[\frac{9}{16\pi^2} P^2 (Q_1 + Q_2)^2 \left(\frac{1}{R_1} + \frac{1}{R_1'} + \frac{1}{R_2} + \frac{1}{R_2'}\right)\right]^{1/3} \quad (1)$$

where:

D = approach of body centers (in.);
K = constant determined from elliptic integral tables;
P = applied compressive load (lb.);
$Q = \frac{4(1-\mu^2)}{E}$;
μ = Poisson's ratio;

E = Young's modulus (p.s.i.);
R = major radius of curvature (in.);
R′ = minor radius of curvature (in.);
1 denotes primary convex body (wheat grain);
2 denotes secondary convex body (steel).

The use of this equation requires that eight fundamental assumptions be satisfied, which have been listed in detail by Kozma and Cunningham (9). Among these assumptions, the contacting bodies are infinitely large and their radii of curvature are large when compared with the dimensions of the contact area; the material is both homogeneous and isotropic; Hooke's Law holds and the loads applied are static.

Morrow (11) has discussed these assumptions as applied to agricultural products, and an example for satisfying the assumption of a semi-infinite body has been given by Morrow and Mohsenin (12). To evaluate an apparent elastic modulus for wheat grain, these assumptions were considered valid and equation 1 was simplified to yield an expression for calculation of the modulus.

Since the wheat grain was loaded on the curved surface at the point of maximum height, its radii of curvature are, by approximation, as shown in Fig. 2.

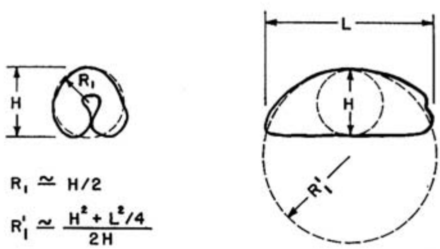


Fig. 2. Estimation of the radii of curvature of wheat grain.

If the second body is steel with $E = 30 \times 10^6$ p.s.i., Q_2 is approximately zero and equation 1 can be simplified into

$$D = \frac{K}{2}\left[\frac{9\ P^2\ (1-\mu_1^2)^2}{\pi^2\ E^2_1}\left(\frac{1}{R_1}+\frac{1}{R_1'}+\frac{1}{R_2}+\frac{1}{R_2'}\right)\right]^{1/3} \qquad (2)$$

Substituting the value $K = 1.303$, determined from the geometric properties of the contacting bodies (9) into equation 2 and solving for $E = E_1$ yields

$$E = \frac{0.5\ P\ (1-\mu^2_1)}{D^{3/2}}\left(\frac{1}{R_1}+\frac{1}{R_1'}+\frac{1}{R_2}+\frac{1}{R_2'}\right)^{1/2} \qquad (3)$$

In the present case, the radii of curvature of the flat steel plate are infinite, so that

$$\frac{1}{R_2} = \frac{1}{R_2'} = 0$$

and equation 3 is reduced to

$$E = \frac{0.5\ P\ (1 - \mu_1^2)}{D^{3/2}} \left(\frac{1}{R_1} + \frac{1}{R_1'} \right)^{1/2} \tag{4}$$

II. Uniaxial Compression by Means of a Smooth Spherical Indenter

Whole grains, crease side down, were fixed as before to a metal plate. The compressive load was applied by means of a steel spherical indenter having a 0.065-in. diameter.

Again, Hertz's solution for contact stresses of two elastic convex bodies can be applied to yield the following equation for calculation of the apparent elastic modulus of the grain.

Under these conditions,

$$\frac{1}{R_2} = \frac{1}{R_2'} = \frac{2}{d_2}$$

where d_2 is the diameter of the steel spherical indenter. Substitution of d_2 into equation 3 yields

$$E = \frac{0.5\ P\ (1-\mu_1^2)}{D^{3/2}} \left(\frac{1}{R_1} + \frac{1}{R_1'} + \frac{4}{d_2} \right)^{1/2} \tag{5}$$

III. Uniaxial Compression by Means of a Cylindrical Indenter

In this series of tests the wheat grains, prepared as before, were subjected to concentrated compressive loads by means of a cylindrical indenter with a 0.016-in. diameter.

The original solution for evaluation of stress-strain relations for semi-infinite bodies loaded by cylindrical rigid indenters was proposed by Boussinesq (14). Later, this solution was expanded by Timoshenko and Goodier (15). According to this solution, when a rigid indenter in the form of a circular cylinder is pressed against the plane boundary of a semi-infinite elastic solid, while the displacement, D, is constant over the circular base of the indenter, the distribution of pressures, q, is not constant and its intensity is given by the equation

$$q = \frac{P}{2\pi a (a^2 - r^2)^{1/2}} \tag{6}$$

where:

P = load on the indenter (lb.);
a = radius of the indenter (in.);
r = distance from the center of the circle on which the pressure acts (in.).

Thus, the pressure is smallest at the center ($r = o$) and it becomes infinite at the boundary of the same area ($r = a$).

The modulus of elasticity of the compressed material, E, is given by the equation

$$E = \frac{P\,(1-\mu^2)}{2aD} \qquad (7)$$

where:

D = displacement of the indenter;
μ = Poisson's ratio.

As in the case of Hertz's solution, the use of equation 7 requires that the fundamental assumptions of elasticity, homogeneity, semi-infinite body, etc., be valid.

IV. Uniaxial Compression of Core Specimens between Two Parallel Flat Plates

Grains were shaped at both ends so that core specimens were obtained. These were fixed in a vertical position to a steel plate with the Duco cement. The upper area was then cut parallel to the plate by means of an AO Spencer 860 sliding microtome. A normal compressive load was applied by means of another parallel flat plate.

In this case, the magnitude of unit contraction of the core specimen (ε) under normal compressive stress (σ) is given, according to Hooke, by the equation

$$E = \frac{\sigma}{\varepsilon} = \frac{P/A}{D_e/H} \qquad (8)$$

where:

E = Young's modulus (p.s.i.);
P = load (lb.);
A = contact area (sq. in.);
D_e = elastic deformation (in.);
H = initial height of the specimen (in.).

Testing Procedure

The testing procedure in each of the four methods was similar and consisted of:

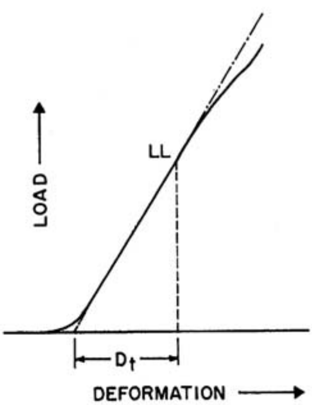
(a) The determination of load-deformation curve to the point of the linear limit, LL, and beyond it. This showed that a considerable portion of the load-deformation relationship was linear.

(b) Load-deformation cycling to a constant load within the linear range of the curve as determined above. Grains were loaded and immediately unloaded to two selected loads within the linear range, and this cycle was repeated three to four times.

The height of each specimen was measured with a micrometer before testing. In methods I and II, the area of indentation was measured under a microscope by means of a calibrated grid pattern disk. In method IV (core specimens), the attainment of full contact area under the applied loads in the cycling tests was ascertained by coating the upper plate before the test with Prussian blue and checking its print on the exposed area after the test. The contact area was measured by means of the microscope.

All tests were performed on the Instron table model with a loading rate of 0.020 in./min. The load cell was calibrated before testing, and its deflection was subtracted from the deformations recorded. The temperature and relative humidity were kept constant throughout the testing at 72°F. (± 1°) and 50% (± 2%), respectively.

The load-deformation curves were similar in shape under all testing methods. They were curved at the initial loading and then turned linear up to a certain point which will be termed the Linear Limit, LL; above the LL load the relation became nonlinear, the deformation increasing at a faster rate than the load (Fig. 3).



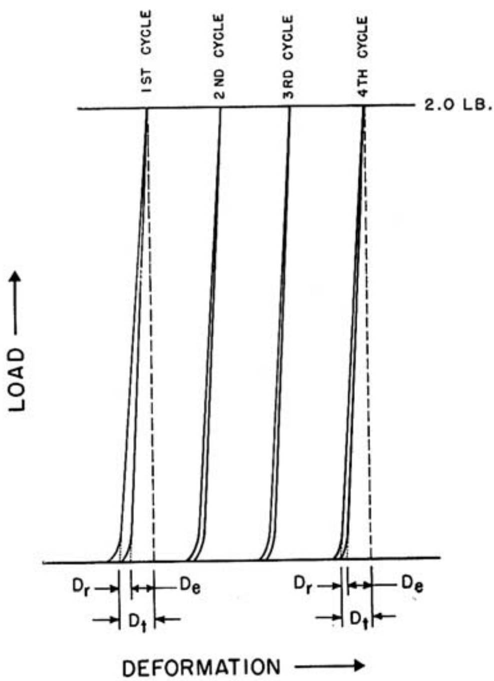


Fig. 3 (left). Uniaxial compression of wheat grain: load-deformation curve.

Fig. 4 (right). Uniaxial compression of wheat grain: cycling to a constant load within the linear range of load-deformation curve.

RESULTS

The loading-unloading curve within the linear range (Fig. 4) showed that part of the total deformation, D_t, is recovered and therefore might be considered elastic deformation, D_e, but part of it is residual and will be termed residual deformation, D_r. Cycling showed no significant change in

TABLE I
EXPERIMENTAL RESULTS FOR UNIAXIAL COMPRESSION TESTS OF SENECA WHEAT[a]

	I Parallel Plate (Whole Grain)			II Spherical Indenter (Whole Grain)			III Cylindrical Indenter (Whole Grain)			IV Parallel Plate (Core Specimens)		
	Mean	S.D.	C.V.	Mean	S.D.	C.V.	Mean	S.D.	C.V.	Mean	S.D.	C.V
Height (H, in.)	0.1174	0.0053	4.5	0.1100	0.0059	5.3	0.1116	0.0050	4.5	0.1543	0.0117	7.6
Contact area (10^{-3} in.^{2})										8.20	1.035	12.6
Load (lb.)	2.0			2.0			1.0			10.0		
Deformation (10^{-3} in.)												
D_e, first cycle	0.530	0.1529	28.9	0.503	0.0650	12.9	0.421	0.0571	13.6	0.840	0.2042	24.3
D_r, first cycle	0.594	0.1742	29.3	0.687	0.3569	52.0	0.322	0.2372	73.7	0.676	0.2411	35.7
D_e, fourth cycle	0.520	0.1281	24.6	0.514	0.0486	9.5	0.403	0.0512	12.4			
D_r, fourth cycle	0.206	0.0999	48.5	0.275	0.1679	61.1	0.064	0.0466	72.8			
Load (lb.)	5.0			4.0			2.0			20.0		
Deformation (10^{-3} in.)												
D_e, first cycle	0.836	0.1300	15.6	0.700	0.0949	13.6	0.569	0.0479	8.4	1.180	0.2024	17.2
D_r, first cycle	0.721	0.1749	24.3	1.037	0.4705	45.4	0.665	0.3985	59.9	0.664	0.1282	19.3
D_e, fourth cycle	0.808	0.1055	13.1	0.718	0.1001	13.9	0.567	0.0536	9.5			
D_r, fourth cycle	0.321	0.0981	30.6	0.394	0.2003	50.8	0.155	0.0789	50.9			
Linear Limit (LL)												
Load (lb.)	11.53	3.7704	32.7	7.08	1.4703	20.8	3.37	0.4678	13.9	28.68	2.9336	10.2
Deformation, D_t (10^{-3} in.)	2.752	0.8111	29.5	2.949	0.8612	29.2	1.843	0.3925	21.3	2.118	0.3181	15.0

[a] Moisture content 10% (d.b.). Deformation rate 0.02 in./min. Each value is a mean of 50 tests for the first three methods and of 20 tests for the last method.

the elastic deformation, whereas the value of the residual deformation gradually decreased, reaching usually a constant value in the third cycle. This value was around 30% of the residual deformation in the first cycle.

It was assumed that ideally the loading curve would be linear even at the origin and that the shape at the initial loading is caused by several factors, such as smoothness of the hull at the point of contact, lack of instantaneous full contact area upon loading, presence of air spaces, soft regions, and other internal irregularities in the material, which are all beyond control. Therefore, each of the deformation values was determined by extrapolating the linear portion of the curve to the point of zero load.

Values of the elastic and residual deformations under two loads within the linear load-deformation relation for each method are shown in Table I. These values were computed for the first and fourth cycles. Also shown in the table are the LL load and total deformation up to this load. Each value is a mean of 50 tests, except for method IV where only 20 specimens were tested.

For calculation of the radii of curvature of the wheat grain in methods I and II, the length (L) and maximum height (H) of 50 grains were measured by means of a micrometer. The mean values of $H = 0.115$ in. (s.d. $= 0.0058$) and $L = 0.260$ in. (s.d. $= 0.014$) were used for the calculation of R_1 and R_1' according to Fig. 2, and these values (0.0575 and 0.1307 in., respectively) were substituted in equations 4 and 5.

Evaluation of an Apparent Modulus of Elasticity for the Individual Wheat Grain

As mentioned before, derivation of the equations for calculating the modulus of elasticity under the four methods (eqs. 4, 5, 7, 8) is based on the assumption that the material under consideration is elastic or that Hooke's law holds. Investigation of the mechanical behavior of the wheat grain in this laboratory and others has shown that the mechanical properties are time-dependent and viscoelastic. Since the load levels used in this work were very low and they were applied for very short times, the elastic behavior of the material was predominant. This was shown by the following:

(a) The load-deformation relation was linear up to a certain load in all four testing methods.

(b) When the material was loaded-unloaded to a low constant load within the linear portion of the load-deformation curve, the deformation was partly recovered and partly residual. Upon cycling, the residual part gradually decreased to small constant values already in the third cycle, whereas the elastic deformation was constant (Fig. 4). Thus, after the third cycle the wheat grain showed no additional plastic deformation and its behavior approached that of an elastic body.

Among the other assumptions made by Hertz (8,9) and Boussinesq (14) as mentioned above, the contacting bodies should be infinitely large, or at least the contact stresses vanish at the opposite end of the body. However, it is impossible to evaluate the validity of this assumption without knowing the stress-strain distribution within the material under compression.

In the original work by Hertz (8) the ratio of the radius of the circle of pressure to the radius of the spherical body under load was considered much less than 1/10. To test the validity of this assumption, the ratio was calculated for methods I and II (Hertz's methods) in two ways: (a) measured under the microscope, as described under "Testing Procedure," and (b) calculated by means of equations available (see ref. 9).

The following shows comparison of this ratio for the two cases where Hertz's method was applied, using the measured and the calculated values of the radius of the circle of pressure for a 2-lb. load.

	Ratio	
	Radius measured	*Radius calculated*
Parallel plate	1/8	1/38
Spherical indenter	1/14	1/15

Although there is a marked discrepancy between the measured and calculated radius of the circle of pressure for the parallel plate compression, the ratios are, on the whole, small enough to justify the assumption as discussed before.

In the case of uniaxial compression by means of a cylindrical indenter (method III), it is possible to evaluate the normal stresses in the Z direction, parallel to the longitudinal axis of the cylindrical indenter, at various points below the surface of a convex body. These normal stresses are shown (15) to be of the form

$$\sigma_z = q\left[-1 + \frac{Z^3}{(a^2 + Z^2)^{3/2}}\right] \tag{9}$$

where the origin of the Z direction is at the surface of the convex body, q is the pressure distribution, and a is the radius of the indenter.

Taking a value of 0.11 in. as the depth of the grain, Z, and 0.008 in. for a, the quantity in the bracket of the above expression approaches zero, indicating that the stress σ_z at this level is approximately zero. The reverse pressure effects caused by the grain's support may be neglected, since the product is supported over a rather large contact area. By symmetry, it can be shown (11) that the stresses produced in the radial and transverse directions are equal in magnitude and therefore will also vanish before the boundaries of the grain are reached. Under these conditions the product can be considered as semi-infinite.

Hertz and Boussinesq also assumed that the material under consideration is homogeneous, which is by no means true in the case of wheat grain. Furthermore, their derivations for calculating the modulus of elasticity require knowledge of Poisson's ratio which is not available for wheat. Poisson's ratio has been found to be about 0.4 for dry shelled corn in this laboratory. There are reasons to believe that the value for wheat is also of about the same magnitude.

From the above considerations, it is realized that one cannot speak of a true modulus of elasticity of wheat grain. The modulus calculated according to equations 4, 5, 7, and 8 can be at best termed the apparent modulus of elasticity, E_a. When the value of Poisson's ratio was required, the modulus was given in terms of kE_a, where $k = 1/(1 - \mu^2)$ and $1 < k < 1.33$. The values of kE_a and E_a for the four methods are shown in Table II. In each case the elastic deformation, D_e, observed under the lowest of the two loads, was used for the calculation, since a better approach was thus obtained to some of Hertz's and Boussinesq's assumptions.

TABLE II
APPARENT MODULUS OF ELASTICITY (AME) FOR WHEAT GRAIN UNDER UNIAXIAL COMPRESSION

WHEAT VARIETY	MOISTURE CONTENT	TESTING METHOD	MEAN AME: E_a (p.s.i.)[a]	
	% d.b.			
Seneca	10.0	Parallel plate (wh. grain)	4.12×10^{5}[b]	(0.25×10^{5})
Seneca	10.0	Spherical indenter (wh. grain)	8.30×10^{5}[b]	(0.25×10^{5})
Seneca	10.0	Cylindrical indenter	1.57×10^{5}[b]	(0.03×10^{5})
Seneca	10.0	Parallel plate (core specimens)	2.30×10^{5}	(0.13×10^{5})
Lyutestsens 62	11–12[c]	Parallel plate (wh. grain) (6)	21.30×10^{5}	(not given)
Gordeiforme 10	11–12[c]	Parallel plate (wh. grain) (6)	28.40×10^{5}	(not given)
Soft red	15.7	Parallel plate (core spec.) (7)	0.46×10^{5}	(not given)

[a] Standard error in parentheses.
[b] The value is for kE_a, where $k = 1/(1 - \mu^2)$ and $1 < k < 1.33$.
[c] Basis not specified.

These values range from 1.6×10^5 to 8.3×10^5 p.s.i., with lower values for the cylindrical indenter and core specimens and higher values where Hertz's method was applied. The low value for the core specimens is predictable, since in this case the structural mechanics of the grain was changed by removal of the ends of the specimen. When the grain is compressed by the cylindrical indenter, a full contact area is immediately reached, as shown by a comparatively small curvature at the initial loading, and the displacement is constant over the circular base of the cylinder. On the other hand, under both of Hertz's methods the area of pressure is gradually increasing with loading and the case is more complex. Since some of the assumptions required by Hertz and Boussinesq cannot be fulfilled for wheat grain, and the present knowledge of the stress-strain distribution within the compressed grain is scant, it is not possible at this stage to conclude which of the modulus values is most accurate.

Two values of the modulus reported elsewhere for parallel plate compression of whole grain (6) and one value for parallel plate compression of core specimens (7) were added to Table II. The first two are larger by one order of magnitude and the value for core specimens is lower than our value. The differences in wheat varieties, moisture content, and experimental techniques used are no doubt the cause of the large discrepancy between the values. Furthermore, the rate of deformation in the first-cited work (6) was much slower than in the present work (by 7 to 16 times), and consequently the time-dependent properties of the material were more pronounced, as evidenced by a relatively low ratio of elastic/residual deformation.

The major purpose of this work was to explore various techniques for better-defined measurements of elastic modulus in wheat grain, justified, as much as possible, by analytical mechanics. The results show that the value for Seneca wheat of 10% moisture content (d.b.) lies in the range of 1.6×10^5 to 8.3×10^5 p.s.i., depending on the method of modulus evaluation. Further investigation is needed to find out which of the testing techniques should be preferred.

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