CFW COLUMN

ENGINEERING

Residence Time Distributions

Leon Levine
Leon Levine & Associates
Albuquerque, NM

Recently, I had the opportunity to hear several presentations and read several articles that discussed residence time distributions. In addition to what was discussed in these instances, I have had, over the years, the opportunity to review and write a number of papers and proposals that covered the same subject. I have come to the realization that these discussions are often incorrect, incomplete, and often misunderstood. In that light, I've decided to devote a few columns, beginning with this one, to the subject.

First we should define what a residence time distribution is and where the concept comes from. The idea of residence time distributions comes from classical chemical engineering literature on the design of chemical reactors. The idea is a fusion of the concepts of probability theory and chemical reaction kinetics. For those interested in the subject, I should point out that the concept is sometimes referred to as population balances, and instead of residence time distribution, the term age distribution is used. There is extensive literature on the subject. The most commonly discussed reference is probably Levenspiel's *Chemical Reaction Engineering*, Wiley, 1962. Another classic book on the subject is Himmelblau and Bischoff's *Process Analysis and Simulation: Deterministic Systems*, Wiley, 1968.

Now, what is a residence time distribution? It is best explained by example. In a batch process, every "particle" of the material being processed, if we neglect filling and emptying of the batch, stays in the process for exactly the same period of time, the total batch time, or residence time of the process. If we were to apply a statistical description of the process, we would say the mean residence time is the batch time, and the standard deviation of the residence time for various particles is zero. That is, there is no dispersion of residence times. If a chemical or physical reaction was occurring, and the material was homogeneous, then the extent of reaction that every particle "sees" would be the same.

Now, if one were to convert this process from a batch operation to a continuous operation, one would want to end up with the same degree of reaction. In order to theoretically do this, we would need to design a process for which every particle stays in the continuous process for the same time as in the batch process. Such a process is not, for simple mechanical reasons, really possible to design. For example, if the batch process is replaced with a long pipe, not every particle stays in the pipe for the same time because of such things as diffusion and drag exerted on the particles by the walls of the pipe. Chemical engineers talk about the idealized continuous process, which exhibit no variance in the residence time of individual particles as a "plug flow" system.

How would one recognize a plug flow system? If the process was operating and, at time zero, one instantaneously injected a small amount of a tracer, say a colorant, and monitored the concentration of colorant at the exit of the process with time, one would see something like Figure 1.

In this figure, all of the tracer exits in an instant of time, the residence time of the system. There is no dispersion in the residence times of various particles. The system exhibits a pure "dead time" response. That is, nothing is seen at the exit until the residence time has been reached, and then all the tracer exits the system in an instant. As stated earlier, no real process exhibits this behavior.

There is a second idealized situation. Imagine a process where there is an equal probability of any particular particle of tracer exiting the system at any instant in time. Chemical engineers call this a perfectly mixed system or a well stirred tank. It turns out that, for this situation, some of the tracer is instantly sensed at the exit of the process and an infinitesimal amount of the tracer takes forever to exit the process. The tracer response for this process would look something like Figure 2.

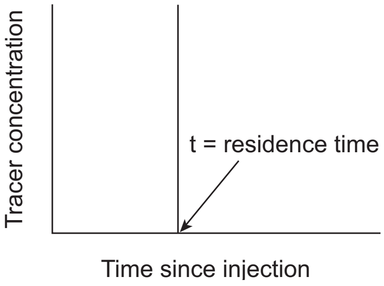

Fig. 1. Response of a plug flow system to injection of a pulse of tracer.

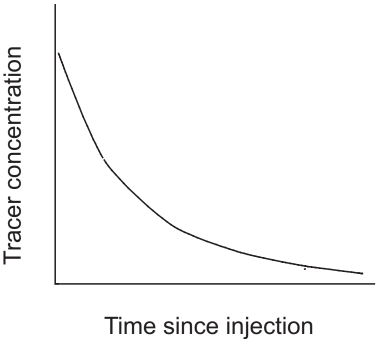

Fig. 2. Response of a well-stirred system to injection of a pulse of tracer.

This response can never really be obtained in a real system. The key thing to note is that various particles of material stay in the process for varying lengths of time. An infinitesimal fraction of material stays in the process for zero time and an infinitesimal fraction stays in the process for an infinite amount of time. The system exhibits no dead time. This means that some material is incompletely processed, say uncooked, and some material is overly processed, say overcooked. Clearly, if the process were some sort of cooking, the quality of the product produced by Figure 1's process would be different than that produced by Figure 2's process. How

doi:10.1094/CFW-53-4-0244

one calculates how big a difference there is will be discussed in a future column. From simply looking at Figure 2 one cannot readily say what the average residence time of a particle is, though it can be calculated. It is clear that there is a dispersion of residence times, or a nonuniform residence time distribution.

As already stated, Figures 1 and 2 represent idealized situations. Any real process would look somewhat different. In fact, any real situation lies somewhere between these two idealized extremes. A real process may yield a response that looks like Figure 3.

The real system illustrated in Figure 3 has some of the characteristics of both Figures 1 and 2. Until the dead time is reached, none of the tracer is detected at the process output. After that time, there is a dispersion of times of exit of the tracer.

The astute reader may have observed that Figures 1–3 represent different distributions of the probability that any particular particle stays in the process. In fact, Figure 3 looks something like a normal distribution. In fact, these curves do represent probability density functions and as such tell us the probability of finding particles that reside in the process between any two times.

One of the common problems that I have observed is a lack of understanding that these tracer responses represent probability distribution functions. As a result, the data is presented in a raw form, such as in Figure 3, and not in a "normalized" form that a statistician would use for describing probability density functions. This leads to a misinterpretation of the physical meaning the curves are providing.

In my next column(s), I will further explain these probability functions and how they should be interpreted and applied to the analysisis of processing problems.

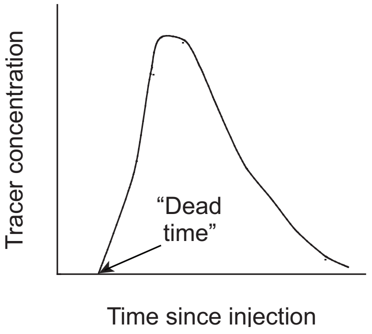

Fig. 3. Response of a real system to injection of a pulse of tracer.

Leon Levine has B.S. and M.S. degrees in chemical engineering and a Ph.D. in agricultural and biological engineering. He is a consultant for the food processing and other consumer-goods industries. Levine can be reached at leon.levine@prodigy.net.