

# Going with the Flow: Computational Fluid Dynamics Simulation of Dough Mixers

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Wheat flour dough is a rheologically complex viscoelastic material, whose unique time-dependent properties depend on the quality of the flour, the available moisture, and the extent to which the dough is mixed. The rheological properties developed in dough during mixing are governed by the rate, amount, and type of deformation applied. As a result, dough mixers have evolved into highly complex geometries that shear, stretch, and fold. In addition, they often have close wall clearances to ensure that there are no regions of ineffective mixing. Changing between mixer types, especially between batch and continuous mixers, is difficult because of the very different flow, shear, extension, and mixing profiles that characterize each one. These differences can have an effect on the mixing time as well as an effect on the gluten structure developed in the dough, which can lead to differences in the final product texture and quality. In industry, determining mixing times and designing mixer configurations is largely done on a trial and error basis.

The farinograph-style twin sigma blade mixers and mixograph-style planetary pin mixers are two common devices used for assessing flour properties during mixing. Both mixers provide empirical measurements related to the rheological properties and energy input during mixing of flour and water dough. They are used to determine flour quality and moisture absorption, despite dissimilar geometries and mixing actions. The results are also frequently used as a guide for determining the mixing requirements of flour in full-size industrial mixers. Because of their importance in the cereal industry, a complete understanding of the flow and mixing action of these particular mixer styles is needed.

- Dough mixers have evolved into highly complex geometries that shear, stretch, and fold.
- The farinograph-style twin sigma blade mixers and mixograph-style planetary pin mixers are two common devices used for assessing flour properties during mixing.
- Computational fluid dynamics simulation has provided a means to more fully understand the flow and mixing action of these particular mixer styles.
- Even though they are often used for similar purposes in the food industry, the flow patterns and mixing action of farinograph and mixograph style mixers are very different.

Computational fluid dynamics (CFD) simulation has provided a means to more fully develop that understanding.

## Simulation Fundamentals, Limitations, and Advantages

CFD simulation software codes have been developed to numerically solve the nonlinear partial differential equations governing all fluid flow, heat transfer, and mass transfer over a discretized domain described as a grid or mesh. The fundamental governing equations include the law of conservation of mass (continuity), the conservation of momentum (Newton's second law of motion), and the conservation of energy (the first law of thermodynamics). These fundamental laws of physics are then supplemented by the various approximations or physical models that have been developed to describe physical phenomena such as those typically seen in various food engineering applications, which include turbulent flow models, porous media and multi-phase models, chemical reaction kinetics models, and non-Newtonian fluid models (19). In addition, CFD simulation techniques have been developed to more easily handle complex geometries and moving parts. These techniques include unstructured meshing, adaptive remeshing, rotating reference frames, multiple frames of reference, sliding meshes, and mesh superposition.

In order to make the best use of CFD simulation, it is important to understand both the strengths and limitations of this technology. Because the underlying physics

are complex and the solutions are based on physical models, the solutions are at best as good as the models upon which they are built (25). Therefore, it is extremely important that material property inputs to the physical laws and models, as well as the boundary and initial conditions, be as accurate of a representation of the actual state and behavior of the fluid or system as possible. This is particularly true of the rheological behavior of fluid materials during mixing. For example, shear dependant viscosity and viscoelastic effects can cause major changes in the flow and pressure profiles seen during mixing, causing mixing faults such as caving and rod climbing. However, when the fluid material being mixed is as complex as dough, it may not be possible to obtain an accurate physical model of the rheological behavior of the fluid material. Does that mean that CFD simulation cannot provide useful information about dough mixers? No, what it means is that what CFD will offer in this case is qualitative insight rather than quantitative numbers (25) using appropriate approximations and model fluids.

Even when the material properties and model parameters can be known with great precision, the numerical techniques used in CFD simulation will contain some error because they rely on approximations over a discrete piece of the problem domain. Although this "discretization" error could theoretically be eliminated by using infinitely fine meshes and time intervals, in practice this is not possible. However, it is

possible to limit the effect of discretization error by refining the mesh only in areas of high activity and change, which requires existing knowledge of the expected behavior of the system being simulated and/or use of an adaptive remeshing technique by the CFD simulation code (5). Other numerical errors that could affect the solution's quantitative accuracy include round-off and local lack-of-convergence errors. The affect of these types of error on the solution can be evaluated using multiple grids with increasingly refined meshes. As finer meshes are used, the CFD solutions will converge toward a solution that is independent of the grid used, and therefore represents a final solution to the set of equations used to define the system.

Another issue to be kept in mind is that most of the physical models used with the fundamental governing equations are based on assumptions about the system that may not hold true everywhere in the flow process. Other assumptions are frequently needed to be made when the boundary and initial conditions are not known precisely. Finally, simplifications are frequently required in order to reduce the complexity of the simulation problem to a manageable level.

With all of these factors contributing to the uncertainty of the meaning and accuracy of the results from CFD simulation, it quickly becomes clear that simulation is not a replacement for experimentation (25). Some level of experimental validation and/or knowledge of the typical or expected behavior of the system being simulated based on experience or theory is required. This information is needed in order to make decisions about the appropriateness of the assumptions and simplifications used in setting up the simulation, as well as to evaluate whether or not the CFD simulations are providing realistic results.

In spite of the limitations listed above, CFD simulation has been shown to offer several unique advantages over experiment-based approaches, such as (19,25):

- The ability to test new equipment designs without the need to build prototypes;
- The ability to study systems where controlled experiments are difficult or impossible to perform;
- The ability to conduct safety studies or accident scenarios without the need to induce unsafe experimental conditions;
- The ability to conduct detailed parametric studies in order to optimize equipment performance; and
- The ability to obtain extremely detailed results that can then be easily postprocessed to calculate a wide range of dependant results.

## GLOSSARY

**Boundary conditions**—Known values of result variables on the edges or at specific points of the flow domain.

**Convergence**—Reduction of the difference between iterative numerical solutions below a preset minimum.

**Discretize**—Break down the flow domain into small pieces for numerical solution.

**Dispersive mixing**—The breakup and redistribution of bubbles, immiscible droplets, or cohesive clumps of material due to mixing.

**Distributive mixing**—The rearrangement of a concentrated component in a mixture to form a random or even distribution due to mixing.

**Flow domain**—The regions of the problem geometry where a solution is to be calculated.

**Governing equations**—The nonlinear partial differential equations governing all fluid flow, heat transfer, and mass transfer.

**Gradient**—The variation of a variable in space (examples: concentration and velocity gradients).

**Mesh**—A way to visualize the discretized flow domain.

**Post-processing**—Analysis and visualization of numerical simulation results.

**Shear thinning**—Materials with a lower apparent viscosity as the shear rate increases.

**Viscoelastic**—Materials with the ability to partially return to their original shape after a force is applied within the time frame of the test.

The initial investment in computer hardware, CFD software, and personnel training as well as the man hours needed to set up and validate a process CFD simulation can quickly be offset if there is a need for the unique advantages of CFD simulation listed above.

## Approaches to CFD Simulation of Mixing

While good mixing is critical to a wide range of processes and industries, it is also an extremely difficult system in which to perform controlled experiments. Consequently, it has been a target for CFD simulation study (7). While motionless or static mixers such as Kenics static mixers (Chemineer, Inc., Dayton, OH) are used in some applications, most mixers involve one or more moving parts that make the flow domain time dependant. In some cases, the time dependence can be removed by changing the frame of reference to that of the moving part, such as using a rotating reference frame in the case of a paddle in a cylindrical vessel (11,12). CFD simulation approaches that have been developed to account for the time dependant motion of the mixing element include dividing the problem domain and using multiple reference frames for the rotating parts and the stationary parts (17), using a sliding mesh around nonintermeshing rotating parts (18), and superimposing the rotating parts on the stationary vessel grid (mesh superposition or fictitious domain method) (1,2,4).

Dough mixing provides an especially interesting mixing system to study using CFD simulation because the mixing is performed not only to homogenously distribute all ingredients, but also to provide energy input in order to develop the gluten. However, rheological models that have been shown to capture aspects of the viscoelastic nature of dough cause convergence problems in CFD simulations. Therefore, those using CFD simulation to study dough mixing have taken two primary approaches. The first is to use simplified two-dimensional (2D) mixing geometries to study the effects of complex rheological properties similar to those of dough on mixing flows. The second approach is to use simple Newtonian fluid models to study the flow and mixing patterns in realistic dough mixing geometries.

## Approach I: CFD Simulation of Complex Dough-like Rheological Behavior in Simplified Dough Mixing Geometries

The complex rheological behavior of dough can generally be characterized as shear thinning and viscoelastic. In order to systematically study the effect of shear thinning and viscoelasticity on mixing flows in a simplified dough mixer, Connelly & Kokini (11) used a representative series of fluid models that included constant viscosity and shear thinning models, as well as constant viscosity-viscoelastic and shear thinning-viscoelastic models. The constant viscosity models had identical viscosities,

while the steady shear viscosity profiles of the shear thinning fluid models were nearly identical and of the same magnitude as the constant viscosity models in the shear rate range of interest. Their mixing geometry consisted of a simplified model 2D mixer consisting of a paddle in a rotating cylindrical barrel, which is based on the geometry of the 2-inch Readco Continuous Processor (Readco Manufacturing Inc., York, PA). CFD simulation was used to generate flow and pressure profiles (12), which were in turn used to track the movement of infinitesimal material points and lines in the flow over time in order to evaluate the effectiveness of the mixing (11). The results showed that viscoelasticity imparted asymmetry to the flow and pressure profiles, especially near the mixing paddle tips, which increased the effectiveness of the distributive mixing. In contrast, while shear thinning greatly reduced the pressure buildup in the mixer, it also increased the size of the regions where material was rotating and not elongating, thus reducing the overall effectiveness and efficiency of the mixing.

A 2D study comparing the mixing ability of a single paddle versus a twin paddle mixer design by Connelly & Kokini (8) using a representative shear thinning dough-like rheological model showed the much greater mixing effectiveness of the twin paddle design, as seen in its ability to distribute a cluster of noncohesive material points. The cluster distribution results show the stringing out and reforming of the cluster in the single paddle mixer along the streamlines, while the twin paddle results were closer to an ideal random distribution. Figure 1

compares the mean of the normalized length of stretch for infinitesimal material lines originally randomly distributed throughout the flow domains during mixing in the two mixers. It shows that the twin paddle mixer produces a logarithmic increase in the length of stretch, which is a necessary requirement for effective distributive mixing. In contrast, the single paddle mixer values stagnate due to the fact that the particles remain stuck on their streamlines.

A group at the University of Wales (3,6,14,23) combined CFD simulation and experimental results in simplified model mixers in order to better understand dough kneading and improve performance of industrial dough mixers. In particular, they studied the wetting and peeling of dough and other liquids on solid surfaces and presented new methods to model the free surfaces. The CFD simulations included viscoelastic fluid models with both constant viscosity and shear thinning behaviors in the fully filled, moving wall mixer case with concentric, eccentric, and double eccentric rotating cylinder model stirrers. They found the asymmetrical stirrer positioning of a single stirrer to provide the best mixing of viscoelastic fluids. Their experimental validation techniques included laser scatter technology, laser Doppler anemometry, and a video capture technique to determine velocity profiles and peeling stresses in a prototype industrial mixer with the cylindrical bowl and stirring rods fashioned from Perspex. The results showed close agreement between simulation and experimental flow fields and free surface profiles.

## Approach II: CFD Simulation of Mixing in Realistic Dough Mixers Using Newtonian Fluid Models

One of the first attempts at simulating the complex flow patterns in more realistic dough mixer geometries was reported by Jongen and coworkers (15,16). They carried out 2D numerical simulations of several batch dough mixers, including a planetary pin (mixograph-style) mixer, double blade (farinograph-style) mixer, interacting twin screw mixer, and a simplified Eberhart spiral mixer, with a Newtonian fluid model using the finite element method (FEM) CFD package Fidap (Fluent, Inc., Lebanon, NH). The movement of the mixing elements was taken into account by using a CFD technique that superimposes the rotating parts on the stationary vessel grid. They evaluated the distribution of shear and elongational flow using a normalized flow-type parameter as well as following the paths of a limited number of material points. Initial comparisons of the specific volume change (oven rise) between proofed and baked crispy rolls mixed in the four dough mixers compared with the space and time averaged normalized flow-type parameter suggested that a deformation combining shear and elongation, as done in the planetary pin mixograph-style geometry, was the most effective at producing oven rise. Another similar three-dimensional (3D) CFD study, which also considered shear thinning fluid behavior, found model rotating pin mixers to provide the highest level of elongation and model twin z blade mixers to provide the highest strain rates, while shear thinning fluids generated larger stagnant zones than did the Newtonian fluids (27).

More recently, realistic 3D FEM numerical simulations of the farinograph (C.W. Brabender, S. Hackensack, NJ) (7,9,10) and the mixograph-style ReoMixer (Reologica Instruments, Lund, Sweden) (7,13) have been done using the CFD simulation package Polyflow (Fluent, Inc., Lebanon, NH). The farinograph has two nonintermeshing sigma blades where the fast (right) blade turns at 93 rpm counterclockwise and the slow (left) blade turns at 62 rpm clockwise. Because the blades turn at different speeds, two revolutions of the slow blade and three revolutions of the fast blade are required before there is repetition of the relative blade positions. Computer aided design (CAD) representations of the blade geometries were provided compliments of C.W. Brabender Instruments, Inc. and transformed into meshes. Figure 2A shows the left (slow) blade mesh of 6,232 tetrahedral elements and the right (fast) blade mesh of 6,166 elements superimposed on the bowl (41,860 hexahedral elements). The time between blades position simulations was

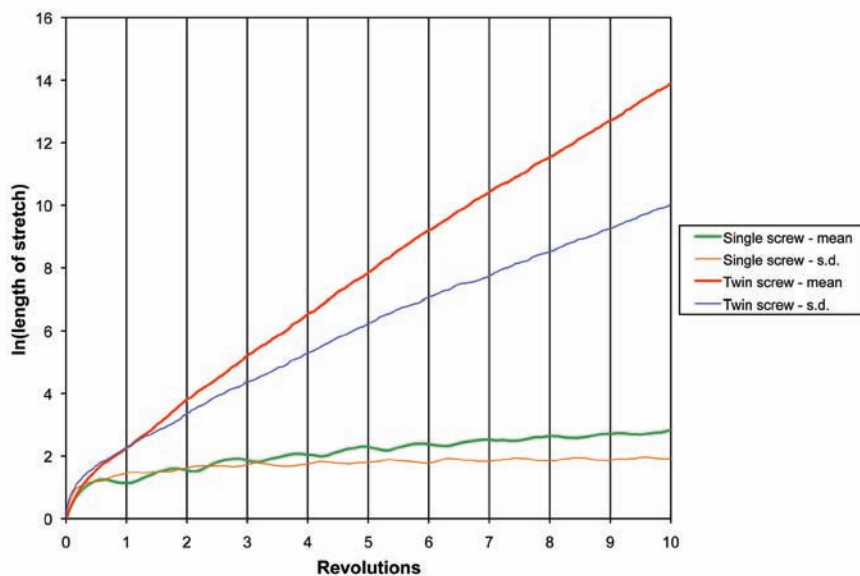


Fig. 1. Comparison of the natural logarithm of the normalized length of stretch between the two-dimensional single- and twin-blade mixers (8, p. 967, © 2007, with permission from Elsevier).

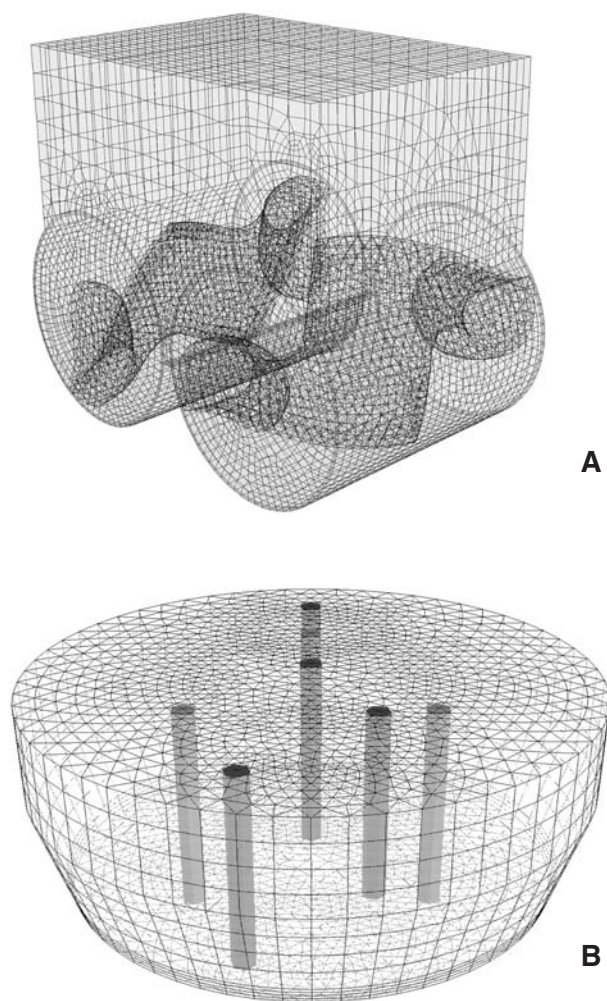
0.027 seconds, giving a total of 72 positions per blade cycle with  $10^\circ$  between positions for the slow blade and  $15^\circ$  between positions for the fast blade.

A different CFD simulation approach was taken to account for pin motion in the ReoMixer (Reologica Instruments, Lund, Sweden), which is a revolving planetary pin mixer similar to the mixograph (National Manufacturing, Lincoln, NB) (7,13). The position of the two revolving pairs of planetary pins resets every 2.73 seconds after three revolutions of the moving pins around their planetary axis and four revolutions of the main gear that rotates at 88 rpm clockwise. The positions of the pins during mixing have been accurately mapped and modeled by Walker & Hazelton (26), but that model was found to not properly account for the individual pin rotation (13). Therefore, in the CFD simulation, the pin motion was completely accounted for by using a rotating reference frame where the main gear was considered stationary, together with mesh superposition of both the stationary (light

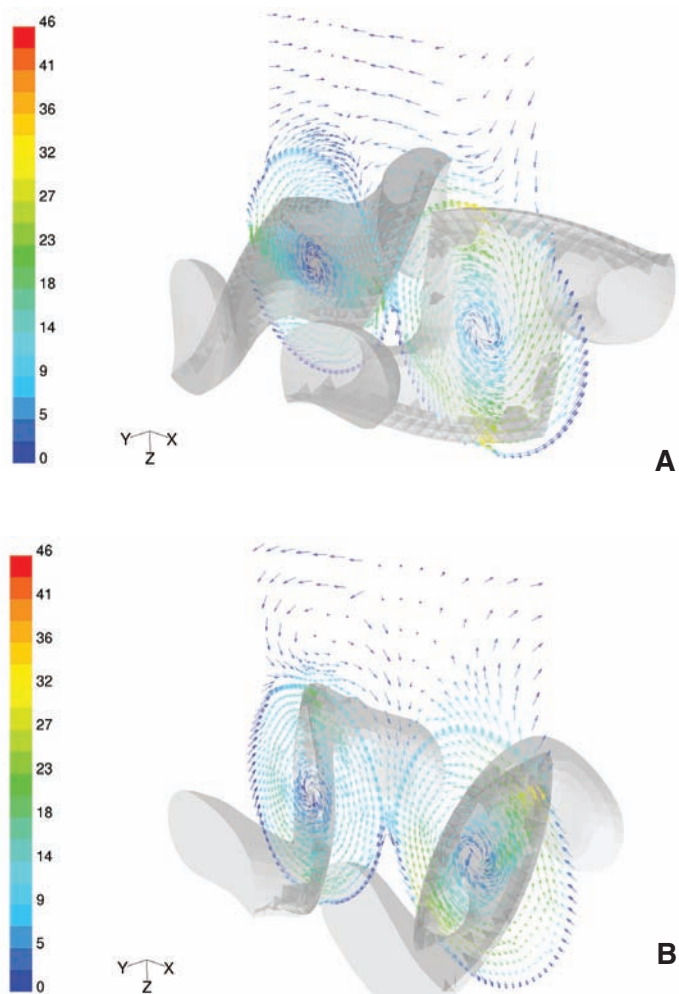
grey in Figure 2B) and revolving pairs of planetary pins (dark grey in Figure 2B). With this approach, the motion of the rotating pairs of planetary pins was confined to two circles in the flow domain that corresponded to the planetary gear rotation. In the meantime, the three stationary pins appeared to revolve around the center of the bowl on a third circle (13) at 88 rpm counterclockwise. This approach also allowed the flow domain mesh in Figure 2B to be designed to better represent the shape of the superimposed pins, while also requiring a much smaller area of the flow domain to be densely meshed in order to properly capture the flow gradients near the pins.

In the CFD simulations, both mixer geometries were represented in the fully filled condition with a simple fluid model that represents the behavior of a viscous Newtonian corn syrup over a geometry reset cycle at the normal operating speed of each mixer. In addition, CFD simulations were also done at two blade positions in the farinograph for a shear thinning 2%

carboxymethylcellulose (CMC) solution and a highly shear thinning 0.11% Carbopol solution using fill and boundary conditions that match those used to generate available experimental laser Doppler anemometry (LDA) data (20,21,22). The effect of varying the rheology in the farinograph was to intensify and shift the areas of high shear rate closer to the blades and increase the size of low shear rate areas of reorientation or rotating flow. The experimental velocity and shear rate results were generally qualitatively in agreement with the CFD simulation results and provided some limited validation of the CFD simulation (10). The complete cycle of Newtonian corn syrup model simulations in the two mixers produced velocity profiles that were then used to calculate the positions over time of a large number of initially randomly distributed massless material points in order to visualize the flow patterns produced in the two mixers. The velocity profiles and massless material point trajectories were also used to calculate measures of dispersive and distributive mixing



**Fig. 2.** Computational fluid dynamics simulation meshes for farinograph-style (A) and mixograph-style (B) mixers (7, © 2007 by Taylor & Francis Group LLC. Reproduced with permission of Taylor and Francis Group LLC via Copyright Clearance Center).



**Fig. 3.** Velocity vectors in a farinograph on a plane across center of bowl ( $y = 4.225$  cm). The arrows show direction and the velocity magnitude is color scaled in cm/s (10, p. 187, © 2006 Wiley Periodicals, Inc.).

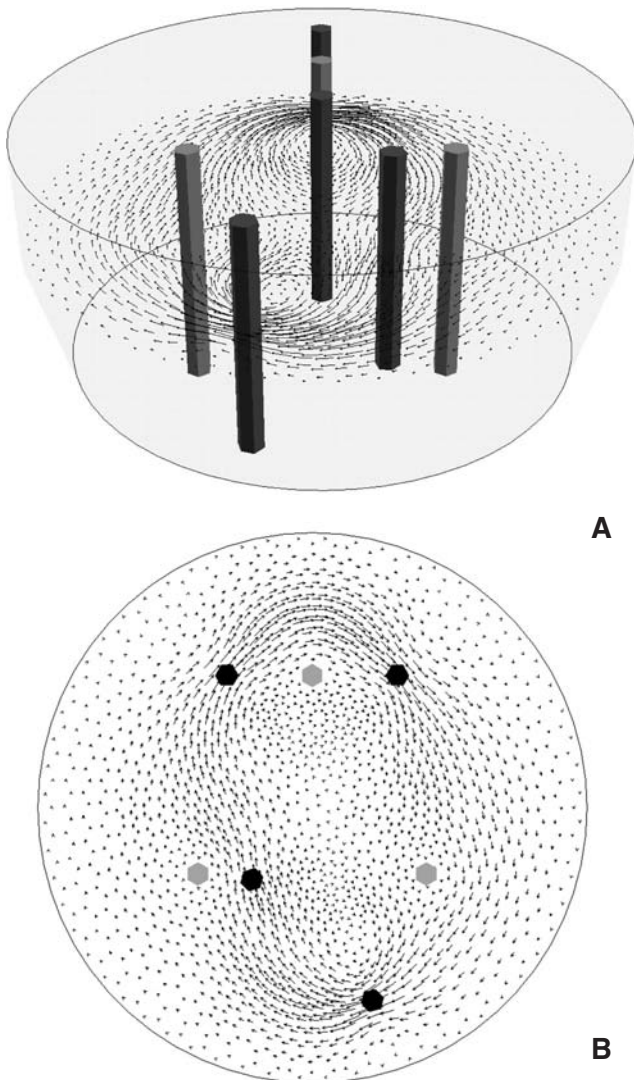
and mixing efficiency, including shear rate and normalized length of stretch (9).

Figure 3 shows the velocity vectors at a plane that cuts across the middle of the farinograph mixing bowl and blades. They show evidence of the three circulation patterns: the primary radial flow pattern of material moving with the blades as well as two slower secondary flow patterns consisting of axial flow due to the forward and backward pumping action of the blades and a slow overall clockwise circulation caused by the speed differential of the blades. The rotational motion imparted by the blades is the dominant component of the velocity in the area swept by the blades. There is a flow reversal point with almost no fluid motion at about the location of the blade axis of rotation, while Figure 3 shows that

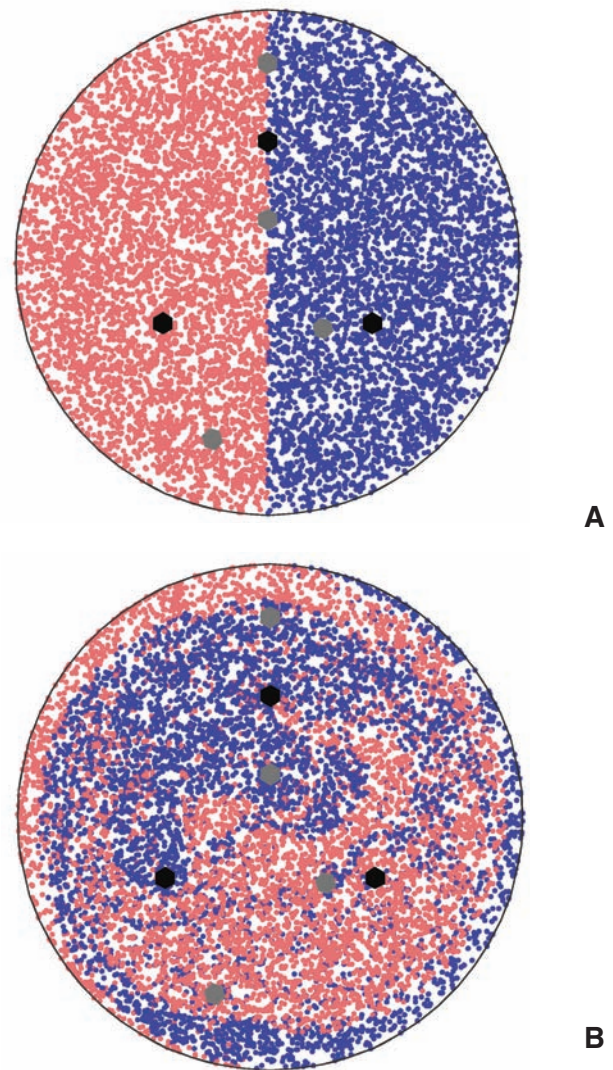
the fastest flow is found roughly 1 to 2 cm away from the walls and near the blade edges. The secondary flow patterns follow the pressure gradients created by the moving blades, which are high on the leading edges of the blades, thus pushing material away and lower on the trailing edges thus drawing material closer. Due to the angle of the blades and the direction of their rotation, the slow blade pumps material toward the front while the fast blade pumps material toward the back. In the  $180^\circ/270^\circ$  position in Figure 3A, material is pushed by the top of the fast blade towards the slow blade, which then pulls the material down. In the  $270^\circ/405^\circ$  position in Figure 3B, the flow pattern in the center is reversed. In addition, the speed differential causes good material exchange between the blades as more material at the

top is pulled towards the fast blade and more material at the bottom is pushed away by the fast blade, even though the blade paths do not intersect. These velocity flow patterns are reflected in the type of flow developed, as indicated by the mixing flow-type index results in Figure 6. While the dominant flow type is shear flow, there are substantial areas with strong elongational character located in between the two blades. In addition, Figure 6A shows how the differential speed increases the deformation of material close to the blade surfaces.

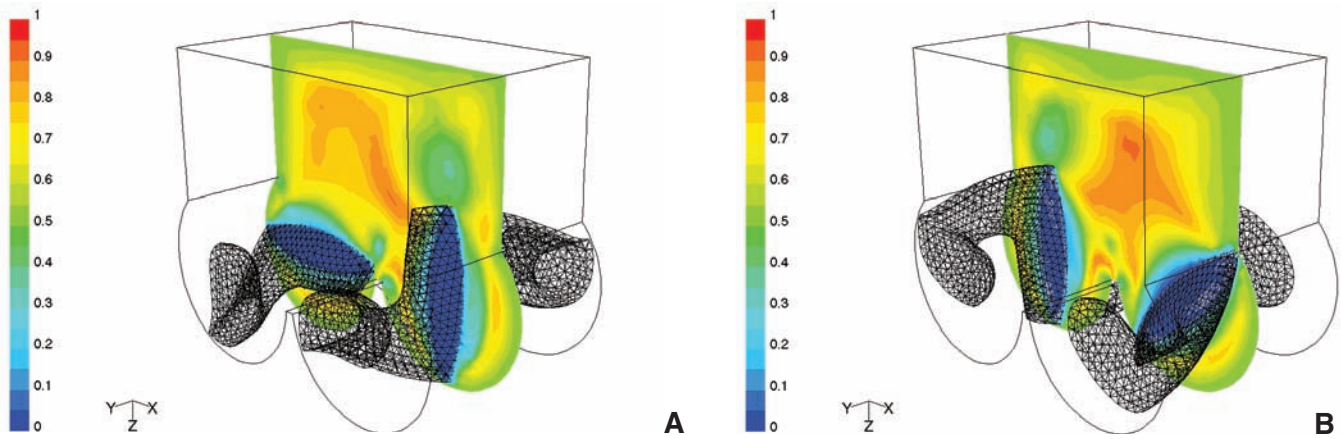
Velocity vectors in the mixograph-style mixer at two positions of the pins are shown in Figure 4 (13). The length of the vector arrows in Figure 4A shows the intense velocity magnitude generated as the material is squeezed between the stationary and



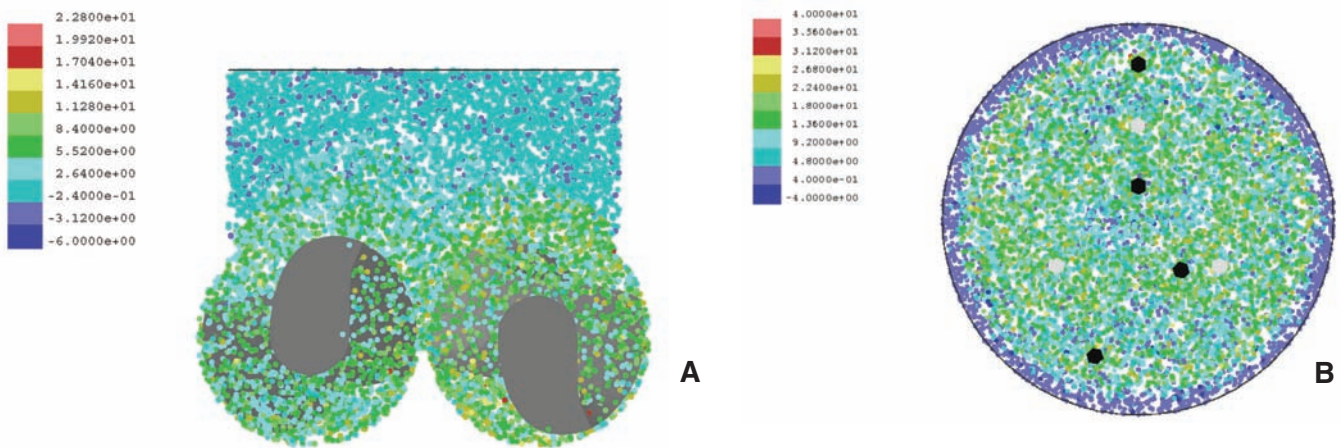
**Fig. 4.** Pin position and standard reference frame velocity vectors at the initial position (A) (7, © 2007 by Taylor & Francis Group LLC. Reproduced with permission of Taylor and Francis Group LLC via Copyright Clearance Center) and after a half geometry reset cycle (B) (two revolutions of the main gear and 1.5 revolutions of the planetary gears) (13). The length of the vector arrows is proportional to the magnitude of the velocity.



**Fig. 5.** Horizontal mixing analysis: data initially (A) and after a one geometry reset cycle (2.7273 seconds) (B) (7, © 2007 by Taylor & Francis Group LLC. Reproduced with permission of Taylor and Francis Group LLC via Copyright Clearance Center).



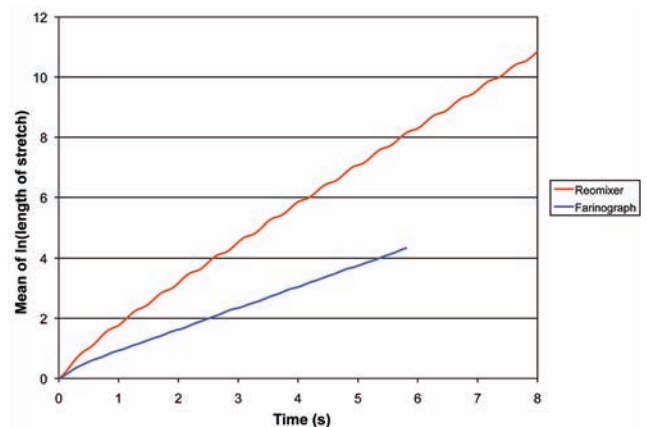
**Fig. 6.** Simulated mixing flow-type index in a farinograph on a plane across center of bowl ( $y = 4.225$  cm) where a value of zero indicates pure rotational flow with no deformation, a value of 0.5 is shear flow, and a value of 1.0 is pure elongational flow (10, p. 192, © 2006 Wiley Periodicals, Inc.).



**Fig. 7.** Length of stretch distribution after three geometry reset cycles for 5.8065 seconds in the farinograph (9, p. 3,390, © 2006 American Institute of Chemical Engineers, with permission from Wiley) (A) and 8.1818 seconds in the ReoMixer (13) (B).

planetary pins near the top, while the change in direction of the arrows seen near the lower left stationary pin indicates a folding motion (7). The greatest stretching was found at the pin positions in Figure 4B because after the moving pins approach and straddle a stationary pin, a highly elongational flow pattern is created as the moving pin pulls away from the stationary pin (13). The effect of these flow patterns on the mixing is demonstrated in Figure 5 (7,13), with the distributive mixing pattern caused by the folding and squeezing clearly evident after two revolutions of the main gear and the corresponding 1.5 revolutions of the planetary pins around their axis, which is a half geometry reset cycle. However, there is little evidence of vertical motion in Figure 4, which led to the finding of little distributive mixing in the vertical direction based on the CFD results. This result is mainly due to the inability of this mixing geometry to generate vertical motion (7,13).

As evident from the preceding discussion, the flow patterns and mixing action of farinograph- and mixograph-style mixers are very different even though they are used for similar purposes in the food industry. Figure 7 shows the relative change in length infinitesimal material lines experience as they are mixed in a Newtonian fluid in the two mixers over three cycles of the geometry back to the initial position (7,8,13). The close clearance scraping action of the farinograph blades ensures that even near the wall with a Newtonian fluid as seen in Figure 7A, the



**Fig. 8.** Arithmetic mean of the natural log of the length of stretch generated over time by 10,000 infinitesimal material lines in the farinograph and ReoMixer (7, © 2007 by Taylor & Francis Group LLC. Reproduced with permission of Taylor and Francis Group LLC via Copyright Clearance Center).

material points are experiencing significant amounts of stretch. The material at the top

with low stretch would not even be present in the normal operation of this mixer (7,8). The elongational nature of the horizontal flow in the mixograph-style mixer causes excellent stretching in the central area as seen in Figure 7B. The low stretched material near the walls in Figure 7B is typical for this type of mixing action with Newtonian fluids, but would peel off into the bulk during normal operation when mixing dough-like viscoelastic materials (3,6,14,23). Figure 8 (7,8,13) shows that the mean length of stretch plotted over time increased exponentially in both mixers, indicating effective mixing for the majority of material points. However, the ReoMixer mean length of stretch increased at a faster rate, which matches the commonly held perception that mixograph-style mixers are more intense than farinograph-style mixers.

### What has been learned and where is the flow going next?

The simplified Newtonian fluid model CFD simulation results for farinograph- and mixograph-style mixers illustrate the wide variation in flow and mixing between these two types of mixers. The results help explain the differences in dough development time, energy input, and dough properties found in dough mixed in these two very different mixers. In addition, the CFD simulation work with dough-like viscoelastic fluid models, while done in highly simplified 2D mixing geometries, still illustrates the effects that the viscoelasticity of dough will have during mixing. As the speed and capacity of computer equipment continues to improve, the effect of numerical error on CFD simulation results will continue to be reduced. However, continued development of CFD simulation techniques to handle free surfaces for complex materials and better techniques to model time-dependant viscoelastic fluids, especially with moving parts, is needed before more quantitatively accurate results for dough mixing processes will be possible. In addition, rheological models that more accurately represent the time-dependant, viscoelastic behavior of dough in the full range of strain rates generated during mixing are also needed. In the meantime, there is still much to be learned about how the mixers we rely on work and can be improved. As the featured results show, CFD simulation provides a qualitative tool to further that study.

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