KINETICS OF BATCH DRYING OF DEEP-BED ROUGH RICE USING DIMENSIONAL ANALYSIS[1]

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ABSTRACT

Drying effects in thick-layer dryers have been investigated by several researchers either by using empirical or mathematical formulations. However, these models cannot be used directly with the commonly known or measurable drying parameters. An attempt has been made to develop a prediction equation for drying time and instantaneous moisture content of wetted rough rice describing thick-layer batch drying with the aid of similitude and dimensional analysis. The parameters involved in this model can be measured both in the laboratory and field without any difficulty. The six dimensionless coefficients were determined for the range of environmental, operating, and product conditions under investigation in this study. The predicted values from the suggested model and the experimental values were in relatively good agreement.

Kinetics of drying in a biological material deal with the characteristics which become apparent during the process, with time in relation to the parameters of the drying agent, namely, temperature, moisture, and speed of its movement.

The moisture content of a biological material such as rough rice is of major concern in its safe storage or proper handling (drying, milling, etc.). To obtain the desired moisture levels, it is often necessary to remove moisture from the harvested materials.

For drying, the material is usually placed in a bulk container and heated air is passed through it. The moisture leaving the grain in response to a vapor pressure gradient between the air and the kernel is picked up by the air and moved out of the grain mass. There is a zone of drying within the grain mass in which each layer in the mass reacts as a single layer fully exposed to its surroundings. A straightforward analysis of a layer within a grain mass is complicated by the fact that the conditions surrounding each layer are continually changing in response to moisture loss by the grain of the previous layers.

Several investigators (1–9) have established models describing the drying effect in thick-layer dryers either by using empirical or mathematical formulations. However, these models cannot be used directly with the commonly known or measurable drying parameters.

An attempt has been made to develop a model on the basis of dimensional analysis and similitude which would predict the drying effect of wetted rough rice with time under the known operating conditions.

DIMENSIONAL ANALYSIS

A schematic of the thick-layer system chosen for the drying investigation is

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shown in Fig. 1. Those parameters believed to be important in this drying system are shown in Table I. Since seed drying is essentially a mass transfer process, the inclusion of the mass parameter for the material in question and the drying air have been taken care of by the drying ratio and relative humidity (ϕ), respectively. Initial, instantaneous, and equilibrium moisture contents of the seed have been grouped to yield the first Pi term (Π_1) and the vapor pressure of the drying air has yielded another Pi term (Π_3). This has been done to make the analysis less complicated and the parameters involved directly measurable.

The rank of the matrix for the basic dimensions is 3. Hence, the number of Pi terms = 8 − 3 = 5. They were: $\Pi_1 = \text{D.R.}$, $\Pi_2 = \gamma/D$, $\Pi_3 = \phi$, $\Pi_4 = V\psi/D$, $\Pi_5 = T_a/T_{si}$.
Hence,

$$\Pi_1 = f(\Pi_2, \Pi_3, \Pi_4, \Pi_5). \qquad [1]$$

It was the object of this test to establish a relation between dependent Pi term, Π_1, and each of the independent Pi terms, keeping, in turn, the other Pi terms constant.

Thus, the four component equations were:

$$\Pi_1 = f(\Pi_2, \overline{\Pi_3}, \overline{\Pi_4}, \overline{\Pi_5}), \qquad [2]$$

$$\Pi_1 = f(\overline{\Pi_2}, \Pi_3, \overline{\Pi_4}, \overline{\Pi_5}), \qquad [3]$$

$$\Pi_1 = f(\overline{\Pi_2}, \overline{\Pi_3}, \Pi_4, \overline{\Pi_5}), \qquad [4]$$

$$\Pi_1 = f(\overline{\Pi_2}, \overline{\Pi_3}, \overline{\Pi_4}, \Pi_5). \qquad [5]$$

These component equations describe the apparent relation between one independent and one dependent variable. Determination of the component equation requires that all but one independent Pi term be held constant while observing the dependent Pi term. In addition, when an independent Pi term is held constant, it should always be the same value (10).

Before a prediction equation may be formulated, the nature of the function must be established. This may be accomplished from analysis of laboratory observations. The resulting relationship is not always simple but, under certain conditions, it may be reasonably direct. In general, the possible methods of formation of general equations from component equations involving 's' Pi terms, discussed by Murphy (10), may be formed either by multiplication or addition of the component equations. For multiplication, the form is

$$\Pi_1 = \frac{F(\Pi_2, \overline{\Pi_3}, \overline{\Pi_4}, \ldots \overline{\Pi_s})\, F(\overline{\Pi_2}, \Pi_3, \overline{\Pi_4}, \ldots \overline{\Pi_s}) \times F(\overline{\Pi_2}, \overline{\Pi_3}, \Pi_4, \ldots \overline{\Pi_s}) \ldots F(\overline{\Pi_2}, \overline{\Pi_3}, \overline{\Pi_4}, \ldots \Pi_s)}{F(\overline{\Pi_2}, \overline{\Pi_3}, \overline{\Pi_4}, \ldots \overline{\Pi_s})^{s-2}}$$

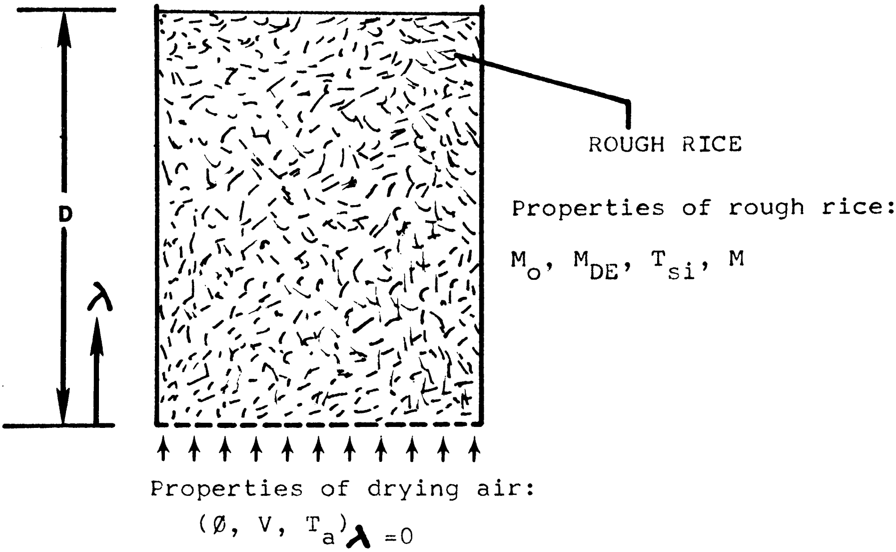

Fig. 1. Schematic of system used in thick-layer investigation.

TABLE I
Basic Parameters in the Thick Layer Drying System

Symbol	Description	Unit	Dimension[a]
D.R.	Drying Ratio= $\dfrac{M_{\psi,\lambda} - M_{DE}}{M_o - M_{DE}}$		[1]
	$M_{\psi,\lambda}$ = Moisture content of seed at time ψ, and distance λ	%, db	[1]
	M_o = Initial moisture content of seed	%, db	[1]
	M_{DE} = Dynamic equilibrium moisture content of seed	%, db	[1]
D	Total depth of seed	ft	L
λ	Depth of the seed layer from the bottom	ft	L
V	Velocity of the air at $\lambda = 0$	ft/min	LT^{-1}
ψ	Elapsed time	min	T
ϕ	Relative humidity of drying air at $\lambda = 0$	%	[1]
T_{si}	Initial absolute temperature of seed	^oR	ϕ
T_a	Absolute temperature of drying air at $\lambda = 0$	^oR	ϕ

[a]ϕ = Temperature, T = Time, and L = Length.

In this, $\overline{\Pi}_2, \overline{\Pi}_3, \ldots \overline{\Pi}_s$ must be the same in each component set. For addition, the form is

$$\Pi_1 = F(\Pi_2, \overline{\Pi}_3, \overline{\Pi}_4, \ldots \overline{\Pi}_s) + F(\overline{\Pi}_2, \Pi_3, \overline{\Pi}_4, \ldots \overline{\Pi}_s) + F(\overline{\Pi}_2, \overline{\Pi}_3, \Pi_4, \ldots \overline{\Pi}_s) + \ldots + F(\overline{\Pi}_2, \overline{\Pi}_3, \overline{\Pi}_4, \ldots \Pi_s) - (s-2)\, F(\overline{\Pi}_2, \overline{\Pi}_3, \overline{\Pi}_4, \ldots \overline{\Pi}_s)$$

However, it is possible for some of the component equations to be combined by multiplication, while others require addition in the formation of the resultant prediction equation. Regardless of whether the resultant prediction equation is formed by multiplication or by addition, a constant term of the form

$$F(\overline{\Pi}_2, \overline{\Pi}_3, \overline{\Pi}_4, \ldots \overline{\Pi}_s)$$

is involved. Obviously this constant may be evaluated from any one of the component equations, and should give the same value from each of the component equations (10).

MATERIALS AND METHODS

To attain the objective, it was necessary to conduct experiments on thick-layer rough rice. This helped in determining the four component equations [2], [3], [4], and [5] which, in turn, yielded the prediction equation for thick-layer drying.

The parameters of these experiments were the temperature and relative humidity of the drying air, air flow rate, elapsed time, initial seed temperature and moisture content, total thickness of the seed column, and the moisture content of a layer of seed in question.

A Starbonnet variety of rough rice was used as supplied by Seed Technology Laboratory, Mississippi State University. The basic apparatus used was an environmentally controlled chamber in which the temperature and relative humidity were regulated (Fig. 2). The inside dimensions of the chamber were 3 × 3 × 3 ft.

Temperature was sensed for the control and recording by an electrical resistance bulb. Gold-grid humidity sensing elements were used for relative humidity control and recording. The chamber provided excellent temperature control well within ±0.5° F. Relative humidity was, in general, controlled within ±1.0%.

Four small Plexiglas seed bins (9 × 8 × 4 in.) provided with fine wire-mesh bottoms were used. Holes (13/32 in.) were provided at a distance of 2 in. from each other on the front wall of each seed bin; this facilitated taking samples of rough rice (6–10 g) at different depths of the seed column with the help of a sample tester during the experiment. Rubber stoppers were used to seal off these holes after the samples were removed. These samples were then accurately weighed.

The entire drying unit was placed in a fully insulated chamber made of plywood with Styrofoam lining. The conditioned air was passed into the plenum chamber of the drying unit by a small centrifugal fan in the environmentally controlled chamber which sent the conditioned air at a constant temperature and relative humidity through a fully insulated duct.

Artificial wetting was used to obtain high moisture seed since freshly harvested rough rice was unavailable at the time of conducting the experiments. About 3 kg

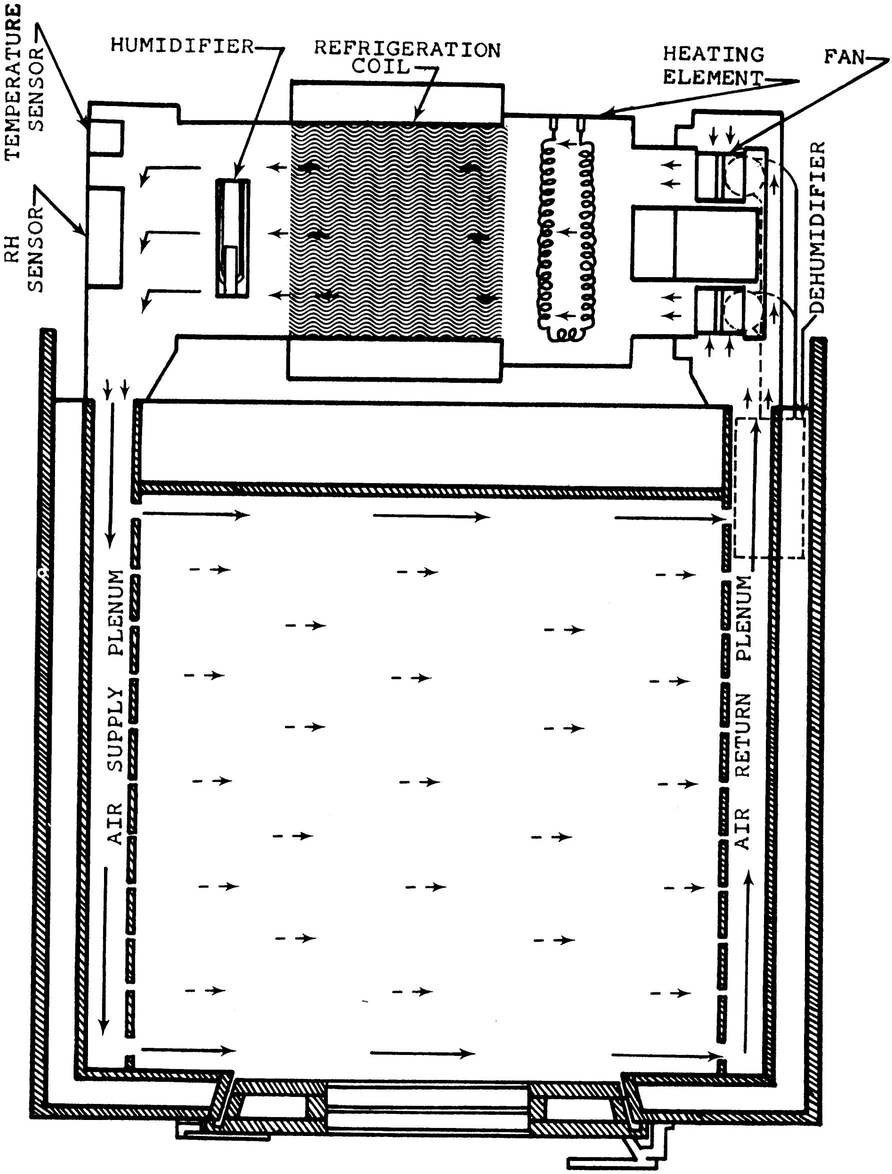

Fig. 2. A cross-sectional view of environment chamber showing air conditioning system and air flow patterns.

of rough rice was soaked in fresh water for about 36 hr. Thereafter, the water was drained off and the grains were spread on a polyethylene sheet in a single layer fully exposed to the atmosphere for sufficient times to reduce the moisture content to desired values within the range of 20–35% (db) as required.

Air flow through the orifice in each of the drying bins was indirectly measured by determining the pressure drop. First, a calibration chart was developed for the pressure drop *vs.* air flow rate in terms of voltage using a hot wire anemometer. The pressure drop in inches of water across the seed bed was measured by a manometer to determine the air flow rate. These values were later checked using a low-velocity anemometer which measures air velocity directly in cm/sec.

The hot-air oven method was used to determine the moisture contents of the samples on a dry basis. Temperature of air inside the oven for whole grain was maintained at 212°F for 72 to 96 hr (11).

The initial depth of 9 in. of seed was tested and the samples were taken at 0.0, 2.0, 4.0, 6.0, and 8.0 in. from the bottom of the drying bin.

Throughout the experiment, the temperature and relative humidity were recorded continuously on a strip chart recorder. The temperature and relative humidity in the plenum chamber were checked by a dew-point hygrometer at an interval of 1–2 hr.

RESULTS AND DISCUSSION

The plotted curves of the component equations and the experimental data are shown in Figs. 3, 4, 5, and 6. Listed in each figure are the component equation,

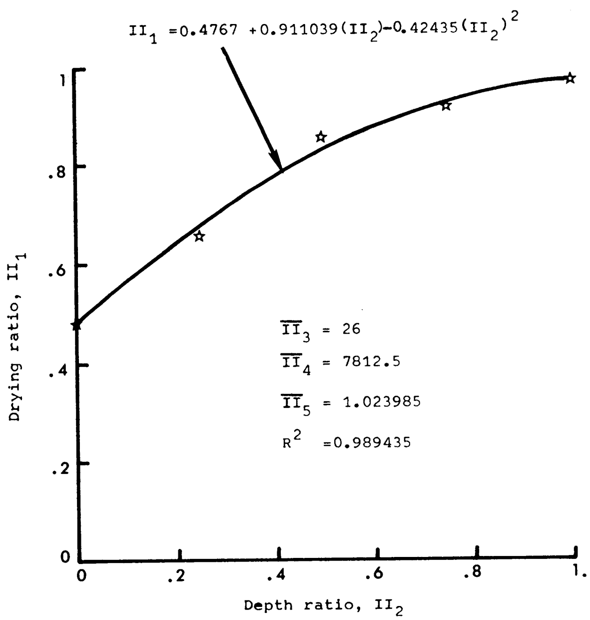

Fig. 3. Drying ratio *vs.* depth ratio.

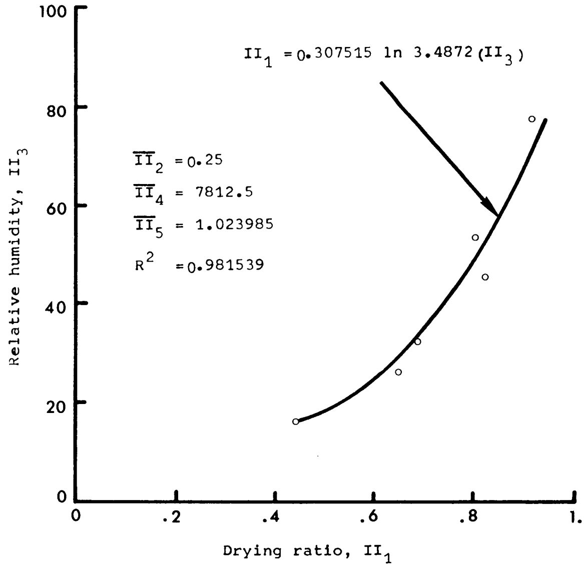

Fig. 4. Drying ratio *vs*. relative humidity.

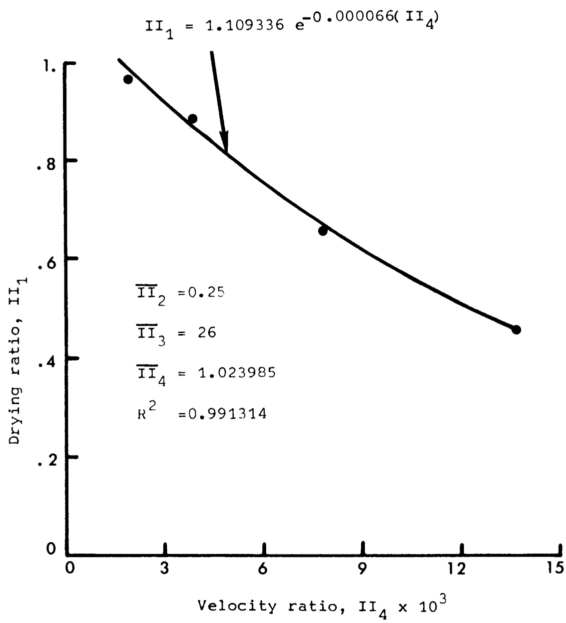

Fig. 5. Drying ratio *vs*. velocity ratio.

coefficient of determination (R^2), and values of the independent Pi terms held constant during the experiment. Data for each test were analyzed statistically (curvilinear regression analysis) to obtain the best fit between dependent Pi term (Π_1) *vs*. the independent Pi terms (Π_2, Π_3, Π_4, Π_5) in turn, keeping the remaining Pi terms constant.

The coefficient of determination has been used here to represent the per cent of the total sum of squares of the experimental data accounted for by the best fit.

The first component equation

$\Pi_1 = F(\Pi_2, \overline{\Pi}_3, \overline{\Pi}_4, \overline{\Pi}_5)$ with $\overline{\Pi}_3 = 26$, $\overline{\Pi}_4 = 7812.5$, and $\overline{\Pi}_5 = 1.023985$ was of the parabolic form

$$\Pi_1 = 0.4767 + 0.911039\,(\Pi_2) - 0.424355\,(\Pi_2)^2 \qquad [6]$$

for five experimental values of Π_2's.

The second component equation

$\Pi_1 = F(\overline{\Pi}_2, \Pi_3, \overline{\Pi}_4, \overline{\Pi}_5)$ with $\overline{\Pi}_2 = 0.25$, $\overline{\Pi}_4 = 7812.5$, and $\overline{\Pi}_5 = 1.023985$ was of the nature

$$\Pi_1 = 0.307515 \ln 3.48726\,(\Pi_3) \qquad [7]$$

for six experimental values of Π_3's.

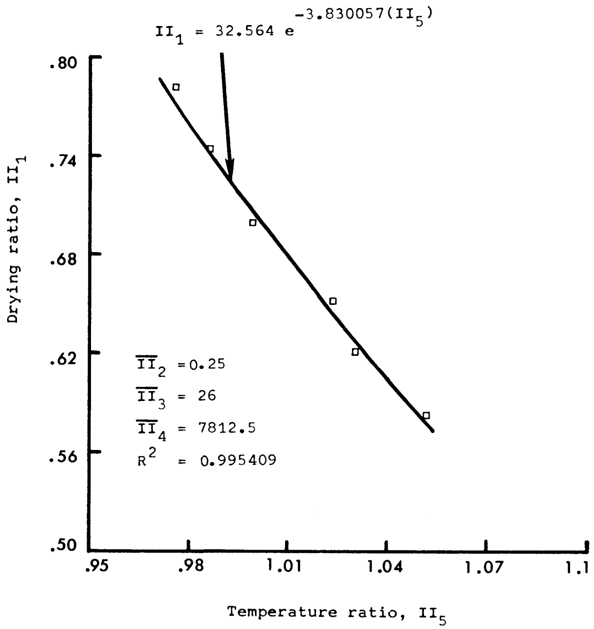

Fig. 6. Drying ratio *vs*. temperature ratio.

The third component equation

$\Pi_1 = F(\overline{\Pi}_2, \overline{\Pi}_3, \Pi_4, \overline{\Pi}_5)$ with $\Pi_2 = 0.25$, $\overline{\Pi}_3 = 26$ and $\overline{\Pi}_5 = 1.023985$ was of the form

$$\Pi_1 = 1.109336\ e^{-0.000066\ (\Pi_4)} \qquad [8]$$

for four experimental values of Π_4's.

The final and fourth component equation

$\Pi_1 = F(\overline{\Pi}_2, \overline{\Pi}_3, \overline{\Pi}_4, \overline{\Pi}_5)$ with $\overline{\Pi}_2 = 0.25$, $\overline{\Pi}_3 = 26$ and $\overline{\Pi}_4 = 7812.5$ was of the nature

$$\Pi_1 = 32.564\ e^{-3.830057\ (\Pi_5)} \qquad [9]$$

for six experimental values of Π_5's.

All four component equations [6], [7], [8], and [9] were obtained for the experimental values based on different replicated drying observations.

Prediction Equation

As mentioned earlier, the prediction equation expresses the dependent Pi term as some function of the independent Pi terms. In the present study, multiplication of the component equations was a reasonably good model for predicting the drying time for wetted rough rice in the range of environmental conditions under investigation. Thus, the prediction equation for Π_1 of the form

$$\Pi_1 = \frac{F(\Pi_2, \overline{\Pi}_3, \overline{\Pi}_4, \overline{\Pi}_5)\ F(\overline{\Pi}_2, \Pi_3, \overline{\Pi}_4, \overline{\Pi}_5) \times F(\overline{\Pi}_2, \overline{\Pi}_3, \Pi_4, \overline{\Pi}_5)\ F(\overline{\Pi}_2, \overline{\Pi}_3, \overline{\Pi}_4, \Pi_5)}{[F(\overline{\Pi}_2, \overline{\Pi}_3, \overline{\Pi}_4, \overline{\Pi}_5)]^3} \qquad [10]$$

was assumed.

The constant value $F(\Pi_2, \Pi_3, \Pi_4, \Pi_5)$ was computed from each of the component equations [6], [7], [8], and [9], with the following results:

From Equation	$F(\overline{\Pi}_2, \overline{\Pi}_3, \overline{\Pi}_4, \overline{\Pi}_5)$	
[6]	0.677938	$\overline{\Pi}_2 = 0.25$
[7]	0.617791	$\overline{\Pi}_3 = 26$
[8]	0.662415	$\overline{\Pi}_4 = 7812.5$
[9]	0.644865	$\overline{\Pi}_5 = 1.023985$

The average value was computed by taking fourth root of the product of the above values and was found to be equal to 0.650366. After substituting the functional values in equation [10], the final form of the drying time model was

$$\psi = c_o \frac{D}{V} \ln \left\{ \frac{(II_1) \quad e^{c_1(II_5)}}{[c_2 + c_3 (II_2) + c_4 (II_2)^2] \ln [c_5 (II_3)]} \right\} \quad [11]$$

where

ψ = drying time, hr.
D = total depth of seed, ft.
V = velocity of drying air, ft/hr, and c_0, c_1, c_2, c_3, c_4, and c_5 are dimensionless coefficients having the following values:

$c_0 = -15151.5152$, $c_1 = 3.8301$, $c_2 = 19.2504$, $c_3 = 36.7901$, $c_4 = -17.1365$, and $c_5 = 0.2867$.

Comparison of Predicted and Experimental Values

Table II shows the observed and the predicted values for drying time in hours for different drying and seed conditions. Additional tests were performed to obtain these observed values. The absolute difference in these two values as seen in Table II ranges from 0.01 to 0.70 hr. This deviation could be attributed to either of the limited experimental points to which the individual component equations were fitted. There could be a possibility of combining the component equations in a different fashion or due to random error.

However, this difference in no way will significantly affect the results in drying operation as far as drying time is concerned. The prediction equation [11] indicates that as the velocity of the drying air increases, the drying time is reduced, while an increase in the total depth of seed will increase the drying time. Also, since the value of c_o in equation [11] is negative, an increase in the drying air temperature will reduce ψ and an increase in the relative humidity and depth ratio will increase the drying time.

The foregoing analysis can be used to estimate the drying time required to dry a material at a particular depth of seed column from initial moisture content (M_o) to a required moisture content (M) provided the drying air temperature (T_a),

TABLE II
Comparison of Predicted and Experimental Values of Drying Time

II_1	II_2	II_3	II_5	Observed(1) (hours)	Predicted(2) (hours)	Difference (1-2) hours	% of Observed
0.9651	0.75	26	1.0240	2.75	3.32	−0.57	−21
0.3015	0.0	26	1.0525	7.00	6.40	+0.60	9
0.9190	0.5	77	1.0240	6.00	6.22	−0.22	−4
0.6022	0.0	26	0.9998	3.00	2.60	+0.40	13
0.1515	0.25	16	1.0240	12.50	13.20	−0.70	−6
0.8069	0.25	53	1.0240	4.00	4.41	−0.41	−10
0.6518	0.25	26	1.0240	4.00	3.99	+0.01	...

initial seed temperature (T_{si}), relative humidity (ϕ), velocity (V) of air, and total depth of seed (D) are known. All these parameters are measurable either in the laboratory or in the field.

One can also predict the moisture content at a particular depth of seed column after a certain interval of time

$$M_{\psi,\lambda} = [(c_0' + c_1' \Pi_2 + c_2' \Pi_2^2) \ln (c_3' \Pi_3) e^{c_4' \Pi_4 + c_5' \Pi_5}] \times (M_o - M_{DE}) + M_{DE} \quad [12]$$

where

c_0', c_1', c_2', c_3', c_4', and c_5' are the dimensionless coefficients whose values were:

$c_0' = 19.2504$, $c_1' = 36.7901$, $c_2' = -17.1365$, $c_3' = 0.2867$, $c_4' = -0.0001$, and $c_5' = -3.8301$.

The importance of the application of the theory of similitude and dimensional analysis can now be visualized in terms of the increased efficiency with which the research in drying of bio-materials can be established. This, in turn, would yield a generalized solution of the existing drying problems. Furthermore, the results may be used directly in predicting the drying conditions for the prototype installations.

The two prediction equations for drying time and moisture content could also be used for nonwetted rough rice and other types of seeds or varieties after determining the six dimensionless coefficients involved for each of them.

The predicted model may be used to find the optimum drying conditions for batch-drying with the aid of nonlinear programming after the objective function and constraints are defined.

Literature Cited

1. BAKKER-ARKEMA, F. W., BICKERT, W. G., and PATTERSON, R. J. Simultaneous heat and mass transfer during the cooling of a deep-bed of biological products under varying inlet air conditions. J. Agr. Eng. Res. 12: 297 (1967).
2. BARRE, H. J., BAUGHMAN, G. R., and HAMDY, M. Y. Application of the logarithmic model to cross-flow deep-bed grain drying. Trans. ASAE 14: 1061 (1971).
3. BAUGHMAN, G. R., HAMDY, M. Y., and BARRE, H. J. Analog computer simulation of deep-bed drying of grain. Trans. ASAE 14: 1058 (1971).
4. BOYCE, D. S. Grain moisture and temperature changes with position and time during through drying. J. Agr. Eng. Res. 10: 333 (1965).
5. CHIEN, K. S. Dimensional analysis of seed moisture movement in deep-bed drying. Unpublished M.S. Thesis, Mississippi State University, State College (1969).
6. HENDERSON, J. M., and HENDERSON, S. M. A computational procedure for deep-bed drying analysis. J. Agr. Eng. Res. 13: 87 (1968).
7. NELSON, G. L. A new analysis of batch grain-drier performance. Trans. ASAE 3: 81 (1960).
8. SPENCER, H. B. A mathematical simulation of grain drying. J. Agr. Eng. Res. 14: 226 (1969).
9. THOMPSON, T. L., PEART, R. M., and FOSTER, G. H. Mathematical simulation of corn drying—a new model. Trans. ASAE 11: 582 (1968).
10. MURPHY, G. Similitude in engineering. Ronald Press: New York (1950).
11. HALL, C. W. Drying farm crops. AVI Pub. Co.: Westport, Conn. (1957).

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