DETERMINATION OF OPTIMAL LEVELS OF SEVERAL EMULSIFIERS IN CAKE MIX SHORTENINGS[1]

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ABSTRACT

Four emulsifiers, monodiglycerides (GMS), sorbitan monostearate (SMS), polyoxyethylene (20), sorbitan monostearate (PSB-60), and glyceryl lacto-palmitate (GLP), were evaluated as emulsifiers for cake mix shortenings. Results obtained from application of response surface methodology approached maximal response when combinations of all four emulsifiers were used. The proportion of each emulsifier to be used in the combination was more precisely estimated by use of a simplex design. This application of the principles of evolutionary operation (EVOP) to experimental designs is believed to be unique. The results appear to justify use of such methods to determine the optimal proportions of components in complex mixtures as in mix formulation.

The selection of an emulsifier system for any particular use has become increasingly complex with the multiplicity of surface-active materials available today. Griffin (1) initiated a method to systematize the selection of nonionics, based on hydrophile-lipophile balance (HLB). The HLB system has been used in many fields, but its application in comestibles awaits a better understanding of the function of surface-active materials in the complex systems found in most food products. Nonetheless, HLB is particularly useful in narrowing the selection from the myriad surfactants available.

In the food industry the choice of emulsifiers is drastically reduced, because only a relatively few are generally recognized as safe (G.R.A.S.) or approved as food additives. Even here, the number of different types available is such that the probability of finding a near-optimum emulsifier combination by trial-and-error methods is minuscule. Some emulsifiers that have been proposed for use in prepared cake mixes are:

Monodiglycerides (GMS)
Glyceryl lactyl fatty acid esters (GLP)
Sorbitan monostearate (SMS)
POE (20) sorbitan monostearate, or polysorbate 60 (PSB-60)
Stearyl-2-lactylic acid (S_2LA)
Propylene glycol monostearate (PGMS)
Stearyl monoglyceridyl citrate
Polyglycerol fatty acid esters

[1]Manuscript received August 26, 1965. Presented at the 48th annual meeting, Minneapolis, May 1963. (Since this work was presented, L. T. Kissell has discussed "Optimization of layer cake formulation by a multiple-factor experimental design" at the 49th annual meeting, April 1964.)

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All of these meet the requirements of the Federal Food, Drug, and Cosmetic Act and are being used, or have been proposed for use, in cakes or cake mix shortenings. Buddemeyer *et al.* (2) compared the first five of the emulsifiers listed and concluded that a four-component system provided greatest improvement. This is reported as a ternary system, inasmuch as these authors considered a 90/10 blend of SMS and PSB-60 as a single emulsifier.

In their binary and ternary systems, Buddemeyer *et al.* (2) used an equal weight of each of the several emulsifiers, still considering the combination of 3 and 4 in a 90/10 ratio as a single emulsifier, for "convenience and simplicity." Though there possibly may be some basis for utilizing SMS and PSB-60 in the fixed ratio, it is hazardous to assume that equal weights of emulsifiers in multiple systems give near optimum results.

Kissell and Marshall (3) have utilized a method originally devised by Box and Wilson (4) to ascertain optimal concentrations of the basic constituents in cake. This paper describes similar techniques used to arrive at optimal levels and combinations of four emulsifiers in cake and cake mix shortenings. The emulsifiers selected are the first four listed above. This was an initial survey experiment to develop a meaningful design for determining the best emulsifier system for cake mixes. The next experiment would also consider the tolerance of the resultant mix to those factors for control of which the manufacturer must depend upon the consumer: 1) extent of mixing, 2) baking temperature, and 3) amount of liquid.

Materials and Methods

Monodiglycerides (sometimes referred to as glycerol monostearate or GMS, although other fatty acids often exceed the stearic acid content) with an alpha monoglyceride content of about 40% have been used in cake mix shortenings since the introduction of prepared mixes. A preference for monodiglycerides of intermediate iodine value (IV), based on the need for maximum storage stability of the mixes consistent with performance, has evolved. The GMS used was derived from tallow by glycerolysis and had a nominal IV of 40 and an alpha mono content of 40–44%.

Glyceryl lactopalmitate (GLP), or glyceryl lactostearate, was introduced into cake mix shortenings in 1956. At the present time, GLP-GMS blends make up the emulsifier system used in over 80% of the cake mixes produced in this country. When GLP is used, it is made from commercial palmitic acid of about 78–90% palmitic acid content; the remainder consists mainly of stearic acid with traces of oleic, myristic, and other acids. GLP used herein was made from palmitic acid with minimum palmitic content of 90% and had the following characteristics: total lactic acid content, 18%; water-insoluble, combined lactic acid, 16%; alpha monoester content (calculated as oleate), 8.1–11.1%; IV, less than 2; free fatty acid, 1.0% max.; and free glycerine, 0.5% max.

Sorbitan monostearate (SMS) and polysorbate 60 (PSB-60) are widely used in the baking industry; about 1954 their use in cake mixes was discontinued because of their uncertain status as food ingredients. This was settled in their favor by the food additive amendment of 1958.

SMS and PSB-60 are nonionic emulsifiers meeting the specifications for these products in Food Additive Orders 21 CFR 121.1029 and 121.1030.

To ascertain definitively the relative functionality of these four emulsifiers, determination of their optimal combination in basic commercial cake mixes (white, yellow, and devil's food) was undertaken. It was thought that if the result from this preliminary experiment was satisfactory, we could then apply the system to any number of the emulsifiers listed and, possibly, include simultaneously the response of the system to the factors included in the "tolerance" test.

A commercial white cake mix without shortening or emulsifiers was used in the basic experiment. Although the precise composition of the mix is not known to us, the formula for white cakes may be represented by that shown below. This formula was used in subsequent testing; it shows the applicability of the method, although the response is not identical.

Formula, Cake Mix "C" — Eggs Added to Batter

Ingredients:	g.	
Cake flour	210	Blend together for 5 min. in dry blender.
Sugar	243	
Salt	6	
Nonfat dry milk	18	
Dicalcium phosphate	3.3	
Monocalcium phosphate	2.2	
Soda	2.85	
	485.35	
Dry ingredients	485.35	Mix in Hobart mixer for 4 min. and put through laboratory finisher.
Shortening and emulsifier	59.75	
	545.10	

Mixing procedure (household mixer):

Add 237 ml. water and 66 g. egg whites. Blend 30 sec. at speed 2. Mix 3 min. at speed 4 while scraping down sides of bowl.

Scale 390 g. into two 8-in. pans.

Bake at 350°F. for 30. min. or until done.

TABLE I
EXPERIMENTAL LAYOUT

Levels: Weight Percent of Shortening and Emulsifier	−1	0	+1
GMS	0	4	8
SMS	0	2.1	4.2
PSB-60	0	0.9	1.8
GLP	0	2.5	5

Number	Level[a]	x_1 GMS	x_2 SMS	x_3 PSB-60	x_4 GLP
1	c d	−1	−1	0	0
2	A c d	1	−1	0	0
3	B c d	−1	1	0	0
4	A B c d	1	1	0	0
5	a b	0	0	−1	−1
6	a b C	0	0	1	−1
7	a b D	0	0	−1	1
8	a b C D	0	0	1	1
9	a b c d	0	0	0	0
10	b c	−1	0	0	−1
11	A b c	1	0	0	−1
12	b c D	−1	0	0	1
13	A b c D	1	0	0	1
14	a d	0	−1	−1	0
15	a B d	0	1	−1	0
16	a C d	0	−1	1	0
17	a B C d	0	1	1	0
18	a b c d	0	0	0	0
19	a c	0	−1	0	−1
20	a B c	0	1	0	−1
21	a c D	0	−1	0	1
22	a B c D	0	1	0	1
23	b d	−1	0	−1	0
24	A b d	1	0	−1	0
25	b C d	−1	0	1	0
26	A b C d	1	0	1	0
27	a b c d	0	0	0	0

[a] An upper-case letter indicates that the variable is at the +1 level; a lower-case letter indicates that the variable is at the intermediate level; and no letter indicates that the variable was not present.

The shortening stock consisted of rearranged lard meeting the following specifications:

Property	*Limits*
Wiley melting point	118° ± 3°F.
Congeal point	106° ± 3°F.
Free fatty acid	0.5% max.
Moisture	0.1% max.
Unsaponifiables and moisture	1.0% max.
Saponification number	195–202
Iodine value	56–72

The shortening stock with the required amounts of the various emulsifiers was heated to 160°F. or until a clear solution was obtained, cooled to 120°F., and added to the dry ingredients as shown in the formula above.

The design for the basic experiment is of the multiple response surface type proposed by Box and Behnken (5). It was chosen as a means of obtaining a measure of the relative effect of the several emulsifiers.

Four independent variables were investigated — the four emulsifiers previously mentioned and those shown below:

% GMS	X_1
% SMS	X_2
% PSB-60	X_3
% GLP	X_4

Twenty-four experimental runs would have sufficed; however, the cakes were baked in duplicate, to obtain an improved estimate of the experimental error associated with the mix make-up and the final properties measured.

Scaling of the independent variables is shown in Table I. The center point of the experimental space, identified as a b c d, is intended to be at, or reasonably near, the expected peak of the response surface of the independent variables. In practice, with six measures of effectiveness, we consider it fortunate if we bracket the peak in each response.

Results

The results of the average of duplicate bakings are shown in Table II. Six dependent variables or measures observed were:

Cake volume, two layers	Y_1
Volume index	Y_2
Symmetry index	Y_3
Grain score	Y_4
Texture score	Y_5
Total score	Y_6

1) Cake volume is the sum of the volume of two layers, in cc., measured by rapeseed displacement.
2) Volume index is the sum of the height of three points — a, b, c — across the diameter.
3) Symmetry index is a measure of the cross-section profile obtained from the previous measure as follows: Symmetry Index, $Y_3 = 2b - (a + c)$.
4) Grain score is the sum of the scores of three experienced judges rating each cake on a scale of 0–5, with 15 as a maximum score for grain — the visual characteristics of cake quality.
5) Texture score is the sum of scores of the same three judges rating each cake for moistness, tenderness, etc. — the kinesthetic qualities observed when the cake is cut, broken, and put in the mouth — and includes flavor effects.
6) Volume in cc. is assigned an arbitrary value based on a scale in which we originally thought that 20 would represent the maximum. The total score, Y_6, is obtained by adding this number to grain and texture scores whereby a "perfect" score would be 50. (Improved formulations and processing techniques since have resulted in cakes scoring over 50.)

TABLE II

MULTIFACTOR RESPONSES OF CAKE QUALITY TO VARIOUS EMULSIFIERS[a]
(Average of two bakings)

	Level	A, % GMS	B, % SMS	C, % PSB 60	D, % GLP	Volume Total, 1	Volume Index, 2	Symmetry Index, 3	Batter Gravity	Volume Score, 4	Grains Score, 5	Texture Score, 6	Total Score, 7
1	c d	0	0	0.9	2.5	1,945	69	3	0.96	2	9	11	22
2	A c d	8	0	0.9	2.5	2,075	72	1	.88	9	6	9	24
3	B c d	0	4.2	0.9	2.5	2,100	75	5	.86	11	5	10	26
4	A B c d	8	4.2	0.9	2.5	2,050	72	2	.88	8	13	12	33
5	a b	4	2.1	0	0	1,885	65	0	.99	2	9	11	22
6	a b C	4	2.1	1.8	0	2,110	73	4	.87	11	9	12	32
7	a b D	4	2.1	0	5	2,025	69	−1	.82	7	11	10	28
8	a b C D	4	2.1	1.8	5	2,115	76	4	.83	11	14	13	38
9	a b c d	4	2.1	0.9	2.5	2,100	75	2	.84	10	7	11	28
10	b c	0	2.1	0.9	0	1,915	66	1	.99	3	5	8	16
11	A b c	8	2.1	0.9	0	2,000	68	1	.95	4	14	12	30
12	b c D	0	2.1	0.9	5	2,180	76	1	.80	14	6	12	32
13	A b c D	8	2.1	0.9	5	2,075	72	4	.87	9	13	13	35
14	a d	4	0	0	2.5	1,915	66	0	.97	3	7	11	21
15	a B d	4	4.2	0	2.5	1,960	66	0	.85	5	12	11	28
16	a C d	4	0	1.8	2.5	2,055	71	3	.89	9	10	12	31
17	a B C d	4	4.2	1.8	2.5	2,125	75	3	.87	11	14	14	39
18	a b c d	4	2.1	0.9	2.5	2,110	74	3	.86	11	7	10	28
19	a c	4	0	0.9	0	1,940	68	0	.98	4	8	10	22
20	a B c	4	4.2	0.9	0	2,050	71	2	.91	8	8	11	27
21	A c D	4	0	0.9	5	2,070	73	4	.84	9	12	14	35
22	a B c D	4	4.2	0.9	5	2,140	74	6	.82	12	14	13	39
23	b d	0	2.1	0	2.5	2,075	72	2	.86	9	10	13	32
24	A b d	8	2.1	0	2.5	1,930	66	−2	.85	3	11	10	24
25	b C d	0	2.1	1.8	2.5	2,000	72	3	.88	6	14	13	33
26	A b C d	8	2.1	1.8	2.5	2,030	74	4	.86	7	14	14	35
27	a b c d	4	2.1	0.9	2.5	2,100	74	2	0.86	10	9	11	30

[a] Numerals with headings (e.g., Volume Total, 1) refer to score numbers; see text.

Only 18 cakes per day could be baked. The entire baking sequence covered 3 days. Although complete randomization of the preparation of all formulations was not followed (cakes within days were prepared in a random sequence), the dependent variable, cake volume, shows no difference in means between days. Since the center point was replicated twice each day, the variation of the point was also checked and no difference between days could be detected. In addition to cake volume, the other dependent variables show a similar difference. (See Table III.)

TABLE III
ANALYSIS OF VARIANCE

	Cake Volume			
	Day 1		Day 2	Day 3
	cc.		cc.	cc.
Σy	36,510		36,900	36,660
Σy^2	74,202,500		76,146,900	74,764,000
	SS	DF	MS	F
Means between days	6,700	2	3,350	3,350/7,456.9 = 0.449[a]
Within days	380,300	51	7,456.9	
Total	387,000	53		

* F 99% (2,51) = 5.05

	Cake Volume at the Center Point		
	Day 1	Day 2	Day 3
	cc.	cc.	cc.
Trial 1	2,100	2,100	2,100
2	2,100	2,120	2,100

Response Surface Form

For each dependent variable, Y, a response surface of the form of 1 (subscript) was fitted by the use of least squares:

$$\begin{aligned} Y_1 = {} & b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 \\ & + b_{11}X_1^2 + b_{22}X_2^2 + b_{33}X_3^2 + b_{44}X_4^2 \\ & + b_{12}X_1X_2 + b_{13}X_1X_3 + b_{14}X_1X_4 \\ & + b_{23}X_2X_3 + b_{24}X_2X_4 + b_{34}X_3X_4 \end{aligned}$$

The resulting equations for the dependent variables are shown in Table IV. Although F tests were used to identify the significant coefficients, all the coefficients are shown in the table. The responses in Table V were calculated with all the coefficients in Table IV.

A better understanding of the interaction of the independent variables can be found through preparation and analysis of two-way tables similar to Table V. Using the equations for Y, we can make substitu-

TABLE IV
RESPONSE EQUATIONS FOR THE DEPENDENT VARIABLES

SCORE	EQUATION
Cake volume	$Y_1 = 2103.3 - 5.8X_1 + 36.3X_2 + 54.2X_3 + 58.7X_4 - 36.3X_1^2 - 29.4X_2^2 - 56.3X_3^2 - 20.6X_4^2 - 47.5X_1X_2 + 45X_1X_3 - 47.5X_1X_4 + 6.3X_2X_3 + 10X_2X_4 - 33.8X_3X_4$
Volume index	$Y_2 = 78.83 - 0.67X_1 + 1.21X_2 + 3.21X_3 + 2.42X_4 - 1.15X_1^2 - 1.33X_2^2 - 2.21X_3^2 - 1.52X_4^2 - 1.38X_1X_3 + 2.25X_1X_3 - 1.62X_1X_4 + 1.00X_2X_3 + 0.50X_2X_4 - 0.12X_3X_4$
Symmetry index	$Y_3 = 2.17 - 0.42X_1 + 0.52X_2 + 1.79X_3 + 0.75X_4 - 0.15X_1^2 + 0.29X_2^2 - 0.71X_3^2 - 0.15X_4^2 - 0.13X_1X_2 + 1.13X_1X_3 + 0.75X_1X_4 + 0.13X_2X_3 + 0.13X_2X_4 + 0.13X_3X_4$
Grain score	$Y_4 = 7.67 + 1.96X_1 + 1.33X_2 + 1.29X_3 + 1.42X_4 + 0.81X_1^2 + 0.50X_2^2 + 2.44X_3^2 + 1.00X_4^2 + 2.63X_1X_2 - 0.25X_1X_3 - 0.25X_1X_4 + 0.13X_2X_3 - 0.50X_2X_4 + 0.75X_3X_4$
Texture score	$Y_5 = 10.50 + 0.33X_1 + 0.42X_2 + 0.96X_3 + 0.88X_4 + 0.15X_1^2 + 0.27X_2^2 + 0.96X_3^2 + 0.33X_4^2 + 1.00X_1X_2 + 0.75X_1X_3 - 0.75X_1X_4 + 0.25X_2X_3 - 0.25X_2X_4 + 0.63X_3X_4$
Total score	$Y_6 = 28.33 + 2.00X_1 + 3.33X_2 + 4.42X_3 + 4.83X_4 - 0.83X_1^2 - 0.46X_2^2 + 1.29X_3^2 + 0.42X_4^2 + 1.25X_1X_2 + 2.25X_1X_3 - 2.50X_1X_4 + 0.25X_2X_3 - 0.25X_2X_4 + 0X_3X_4$

tions for X_1, X_2, X_3, and X_4 as shown and calculate the corresponding value for Y_1, Y_2, etc. An arbitrary definition of "acceptable" can be applied to the dependent variable, and those combinations of independent variables yielding an acceptable Y can be noted. For example:

$$Y_1 = 5.83X_1 + 36.25X_2 + 54.17X_3 + 58.75X_4 - 36.25X_1^2 - 29.37X_2^2 - 56.25X_3^2 - 20.62X_4^2 - 47.50X_1X_2 + 45.00X_1X_3 - 47.50X_1X_4 + 6.25X_2X_3 + 10.00X_2X_4 - 33.75X_3X_4 + 2103.33,$$

where $X_1 = +1$, $X_2 = +1$, and X_3 and X_4 range from -1.0 to $+1.0$

$$X_3 = -1, -0.5, 0, +0.5, +1$$
$$X_4 = -1, -0.5, 0, +0.5, +1$$

Results in calculated cake volumes are shown in Table V.

A further step, useful for improving understanding, is the sketching of contour lines of the surface. Using the same two-way table and rounding the data to three significant digits, we can prepare the con-

TABLE V
CALCULATED CAKE VOLUMES

		$X_1 = 1.0, X_2 = 1.0$				
				X_3		
		−1.0	−0.5	0	+0.5	+1.0
	−1.0	1,803	1,916	1,999	2,054	2,082
	−0.5	1,836	1,940	2,015	2,062	2,081
X_4	0	1,859	1,954	2,021	2,060	2,070
	+0.5	1,871	1,958	2,016	2,048	2,048
	+1.0	1,873	1,952	2,001	2,024	2,017

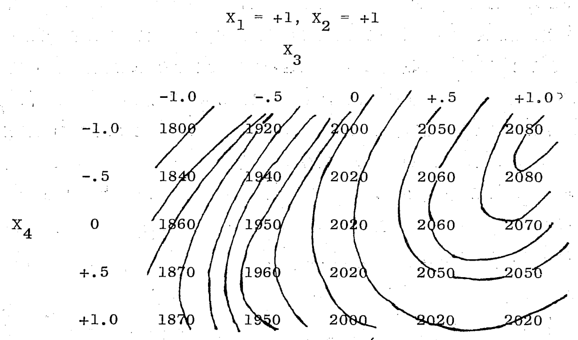

Fig. 1. Surface contours of calculated cake volumes.

tours as shown in Fig. 1. Thus, it is possible to "see" in which direction the surface is changing under various levels of the independent variables.

The "best" product space can be located by constructing a series of two-way tables for each of the dependent variables.

Then by establishing minimum standards for each response term

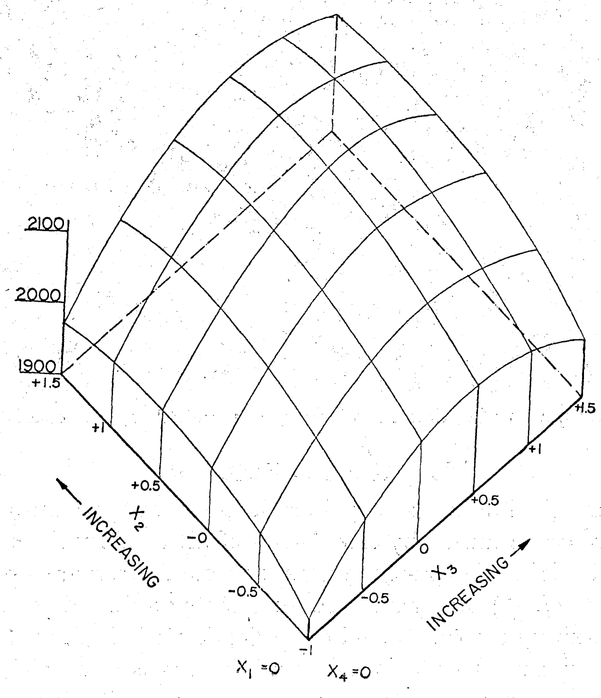

Fig. 2. Calculated cake volume response surface.

and eliminating the unacceptable results in turn, we can arrive at the areas of acceptable formulation suitable for further examination. A minimum value of 2,100 cc. for cake volume would have eliminated all except three areas unsuitable by the other standards. This provides some justification for the use of volume as the sole index of quality in the screening of mix formulations.

A response surface showing graphically the effects of varying X_2 and X_3 while X_1, X_4 are held constant is shown in Fig. 2 and Fig. 3 as suggested by Kissell and Marshall (3).

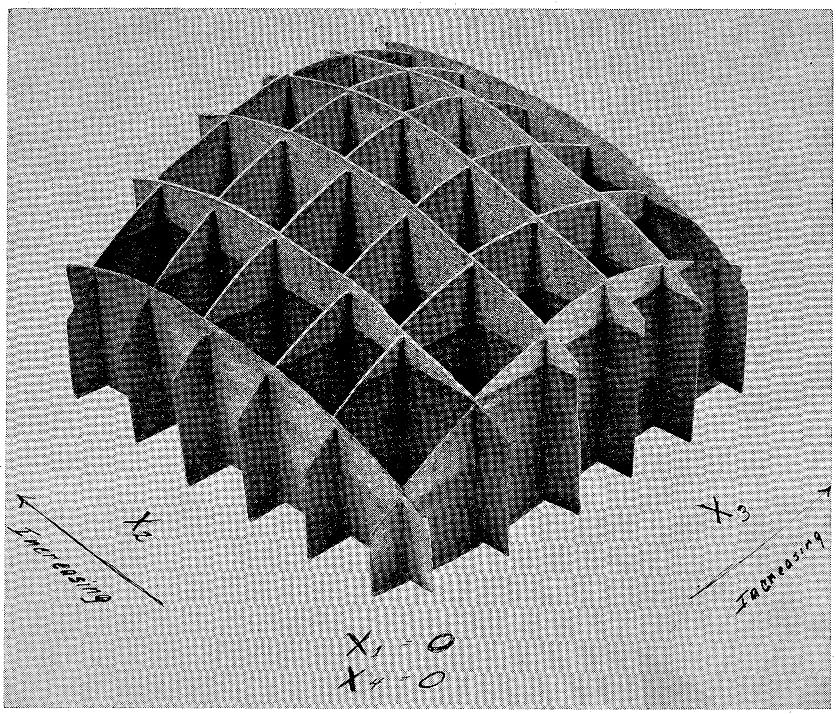

Fig. 3. Determination of optional levels of several emulsifiers in cake mix shortenings.

Experimental Verification

Some insight having been gained into the general form of the response surfaces obtained by calculation of all combinations of X_1, X_2, X_3, and X_4, an optimum-seeking experimental program described by Spendley *et al.* (6) was employed to enable rapid movement to the area of maximum response. The application of simplex designs permits the experimenter to move rapidly across the surface to the vicinity of near maximum response. It does not allow examination of the entire surface or a general understanding of the several responses.

The primary considerations in applying the method are a suitable starting point, and where and when to move from the original experiment. The starting point shown below, where the independent variables produced near-optimal results, was selected. If we accept the dictum to move as often as possible, preferably after each observation, the only other decision is where to move. So long as the scales of the separate factors are such that a unit change of each is of equal interest, the concept of regularity is preserved, and thus the scale length shown was chosen.

	Center Point %	*Scale Length* %
X_1	4	2.0
X_2	2	1.0
X_3	1	0.5
X_4	2	1.0

The coordinates of each observation point in the original set are calculated as follows:

If D_0 is the first set, then the coordinates of each data point in the set can be expressed as:

$$D_0 = \begin{bmatrix} X_1 & X_2 & X_3 & X_4 \\ 0 & 0 & 0 & 0 \\ p & q & q & q \\ q & p & q & q \\ q & q & p & q \\ q & q & q & p \end{bmatrix} \begin{bmatrix} \text{Response} & \\ \text{First observation} & R_1 \\ \text{Second} & R_2 \\ \text{Third} & R_3 \\ \text{Fourth} & R_4 \\ \text{Fifth} & R_5 \end{bmatrix}$$

$$p = \frac{1}{k\sqrt{2}} \left[(k-1) + \sqrt{k+1} \right]$$

$$q = \frac{1}{k\sqrt{2}} \left[\sqrt{k+1} - 1 \right]$$

where k is the number of variables. For k = 4 the scaled original set found by substitution above for p and q is:

$$D_0 = \begin{bmatrix} X_1 & X_2 & X_3 & X_4 \\ 0 & 0 & 0 & 0 \\ 0.93 & 0.22 & 0.22 & 0.22 \\ 0.22 & 0.93 & 0.22 & 0.22 \\ 0.22 & 0.22 & 0.93 & 0.22 \\ 0.22 & 0.22 & 0.22 & 0.93 \end{bmatrix} \begin{bmatrix} \text{Response} \\ \\ \\ \\ \\ \\ \end{bmatrix}$$

The values in D_0 are found by taking the appropriate initial value (found in row 1) and adding to it the unit value of the variable, times either the p or q as indicated in D_0. Thus the coordinates of the points are found as in Table VI.

In actual percentage values, each point in D_0 is:

$D_0 =$

X_1	X_2	X_3	X_4	Response
4.00	2.10	0.90	2.5	R_1
5.86	2.32	1.01	2.72	R_2
4.44	3.03	1.01	2.72	R_3
4.44	2.32	1.36	2.72	R_4
4.44	2.32	1.01	3.43	R_5

From the first set, D_0, a response column R can be filled in with the appropriate observed values for each point. When all the R values in the D_0 are entered, the point with the lowest observed R value is eliminated and replaced by the point reflected through the opposite side of the simplex. The coordinates of this new point are found as follows.

Assume that the lowest observation is R_4 whose X_1, X_2, X_3, X_4 coordinates are identified in row 4. The new value of the X_1 coordinate $X_{1,4}{}^*$ is found as follows (the first subscript will define the variable, the second subscript, the row, and the number of asterisks defines the number of replacements of the point):

$$X_{1,4}{}^* = \frac{2}{k} (X_{1,1} + X_{1,2} + X_{1,3} + X_{1,5}) - X_{1,4}$$

$$X_{1,4}{}^* = \frac{2}{4} (0 + 0.93 + 0.22 + 0.22) - 0.22$$
$$= 0.465;$$

$$X_{2,4}{}^* = \frac{2}{4} (0 + 0.22 + 0.22 + 0.22) - 0.93$$
$$= -0.60;$$

$$X_{3,4}{}^* = \frac{2}{4} (0 + 0.22 + 0.93 + 0.22) - 0.22$$
$$= 0.465; \text{ and}$$

$$X_{4,4}{}^* = \frac{2}{4} (0 + 0.22 + 0.22 + 0.93) - 0.22$$
$$= 0.465$$

Substituting in D_1, the $X_{1,4}{}^*$ for the $X_{1,4}$ values in D_0, we get:

$D_1 =$

X_1	X_2	X_3	X_4	Response
0	0	0	0	R_1
0.93	0.22	0.22	0.22	R_2
0.22	0.93	0.22	0.22	R_3
~~0.22~~	~~0.22~~	~~0.93~~	~~0.22~~	R_4
0.22	0.22	0.22	0.93	R_5
0.465	0.465	−0.60	0.465	$R_1{}^*$

TABLE VI
COORDINATES OF POINTS

Row	Variable/Point X$_1$	X$_2$	X$_3$	X$_4$
1	4% + 0 = 4%	2.1% + 0 = 2.1%	0.9% + 0 = 0.9%	2.5% + 0 = 2.5%
2	4% + 0.93 (2%) = 4% + 0.1.86% = 5.86%	2.1% + 0.22 (1%) = 2.1% + 0.22% = 2.32%	0.9% + 0.22 (0.5%) = 0.9% + 0.11% = 1.01%	2.5% + 0.22 (1%) = 2.5% + 0.22% = 2.72%
3	4% + 0.22 (2%) = 4% + 0.44% = 4.44%	2.1% + 0.93 (1%) = 2.1% + 0.93% = 3.03%	0.9% + 0.22 (0.5%) = 0.9% + 0.11% = 1.01%	2.5% + 0.22 (1%) = 2.5% + 0.22% = 2.72%
4	4% + 0.22 (2%) = 4% + 0.44% = 4.44%	2.1% + 0.22 (1%) = 2.1% + 0.22% = 2.32%	0.9% + 0.93 (0.5%) = 0.9% + 0.46% = 1.36%	2.5% + 0.22 (1%) = 2.5% + 0.22% = 2.72%
5	4% + 0.22 (2%) = 4% + 0.44% = 4.44%	2.1% + 0.22 (1%) = 2.1% + 0.22% = 2.32%	0.9% + 0.22 (0.5%) = 0.9% + 0.11% = 1.01%	2.5% + 0.93 (1%) = 2.5% + 0.93% = 3.43%

and converting these values to percentages we get:

$\mathbf{D_1} =$

X$_1$	X$_2$	X$_3$	X$_4$	Response
4	2.1	0.9	2.5	R$_1$
5.86	2.32	1.01	2.72	R$_2$
4.44	3.03	1.01	2.72	R$_3$
~~4.44~~	~~2.32~~	~~1.36~~	~~2.72~~	R$_4$
4.44	2.32	1.01	3.43	R$_5$
4.93	2.565	0.60	2.965	R$_4$*

Note that the response column of D_1 differs from that of D_0 only in row 4, and the only new point described is 4*, since the R values from D_0 can be substituted in D_1 except for row 4. Now the R values for D_1 are examined, the lowest value is identified, and that point is eliminated as in moving from D_0 to D_1.

With the values for the several variables as the starting point, five cake mixes were prepared and baked as the original set. Then, varying the formula somewhat[3] as indicated above, after eight individual experimental runs consisting of making and baking only one cake mix in each experiment, we arrived at the fomula designated as "E" in Table VII.

This emulsifier system has been evaluated in yellow and chocolate cake mixes of the same manufacturer as the white cake mix. It produces excellent cakes with good tolerance to variation in the amount of water added, baking temperature, and extent of mixing.

Whereas an emulsifier system based on this type of experimentation is suitable for the other basic mixes from the same manufacturer—when another company's cake mix is used, entirely different propor-

[3] Because there were some errors in the calculations we did not follow the design exactly, yet arrived at a very favorable formula very quickly. This formula has been evaluated in yellow, white, and chocolate cake mixes with excellent results.

TABLE VII

SIMPLEX DESIGN CAKE MIX FORMULATION[a]

	GMS	SMS	PSB-60	GLP	VOLUME, 1	VOLUME INDEX, 2	SYMMETRY INDEX, 3	SPECIFIC GRAVITY	VISCOSITY	TOTAL SCORE, 6
	%	%	%	%	cc.				cp.	
1	4.0	2.1	0.9	2.5	2,300	80	4	0.76	3,040	39
2	5.9	2.3	1.01	2.7	2,350	80	4	0.87	3,360	41
3	4.4	3.0	1.01	2.7	2,280	79	2	0.80	3,280	38
4	4.4	2.3	1.4	2.7	2,300	79	2	0.84	3,520	37
5	4.4	2.3	1.01	3.4	2,380	81	3	0.78	3,520	42
A	4.9	2.7	0.6	4.0	2,350	78	3	0.78	3,400	39
B	5.84	3.02	0.915	3.42	2,270	79	5	0.87	3,328	38
C	5.36	2.78	1.37	3.18	2,350	79	2	0.85	3,760	38
D	3.28	2.1	1.1	2.5	2,370	81	3	0.85	3,304	39
E	4.0	0.9	2.5	2.5	2,400	82	5	0.85	3,800	48
F	4.93	2.6	1.22	3.0	2,400	79	4	0.83	3,380	43
G	4.0	2.8	1.11	3.2	2,350	77	4	0.84	3,192	37

[a] Numerals with headings (e.g., Volume, 1) refer to score numbers; see text.

tions of emulsifiers are needed to give near-optimal results. This is to be expected, since the various manufacturers use different flours, sugar-to-flour proportions, and leavening systems, and markedly different amounts of shortening plus emulsifiers. However, what we have presented is a systematic method for arriving at near-optimal combinations of emulsifiers for any given type of cake mix.

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The application of the multiple response surface design to the evaluation of emulsifiers in cakes and cake mixes was suggested by H. Mack Truax, Atlas Chemical Industries, Inc.

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